THEORETICAL APPROACH FOR MEDICAL IMAGE ENHANCEMENT IN LOGARITHMIC REPRESENTATION

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Abstract

The logarithmic image processing (LIP) theory is a mathematical framework that provides a set of specific algebraic and functional operations and structures that are well adapted to the representation and processing of non-linear images, and more generally of non-linear signals, valued in a bounded intensity range. The purpose of this paper is to introduce a new mathematical LIP model focused on theoretical and practical aspects concerning the enhancement of the transmitted medical images and the physical absorption/transmission laws expressed within LIP mathematical framework. First of all the bounded interval (-1,1) is considered as the set of gray levels and we define two operations: addition$\langle + \rangle$ and real scalar multiplication $\langle \times \rangle$. With these operations, the set of gray levels becomes a real vector space. Then, defining the scalar product $(.|.)$ and the norm $||\ .||$, we obtain an Euclidean space of the gray levels. Secondly, we extend these operations and functions for color images. Finally, the experimental results, shown as enhanced medical images, reveal that this method has wide potential areas of impact which may include: Digital X-ray, Digital Mammography, Computer Tomography Scans, Nuclear Magnetic Resonance Imagery and Telemedicine Applications.

Keywords: Medical imagery, image enhancement, logarithmic image processing, real vector space, Euclidean space.

1. INTRODUCTION

Medical images represent, in most cases, a noisy image environment due, primarily, to the limitations placed on X-ray dose, for example. In a large number of situations the medical expert must inspect, visually segment and analyze various objects within a medical image in order to diagnose diseases. This is especially so in breast cancer screening and brain cancer tests. It is generally accepted that due to human factors there are varying degrees of inconsistencies in the diagnostic conclusions, reached by medical experts. In some cases, a false positive diagnostic conclusion is indicated, while in others it is a false negative. Obviously, the only means of reducing the amount of inconsistencies is the use of computer enhanced images. The user interface should allow for proper adjustments so the medical expert (the radiologists) can achieve the preferred image perception in order to come up with a final and correct decision.

For telemedicine applications, particularly, there is a data channel with a limited bandwidth between doctor and patient which may pose a potential bottleneck. This mathematical model compacts the high input dynamic range, potentially reducing the high bandwidth requirement.

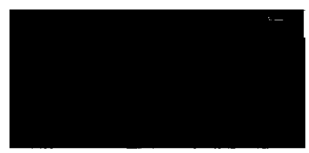
Original digital X-ray pelvis image

Contrast enhanced pelvis image

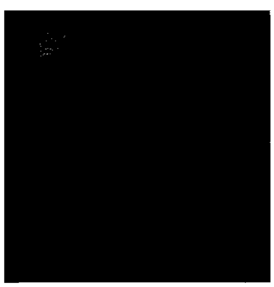
Original digital X-ray femur image

Contrast enhanced femur image

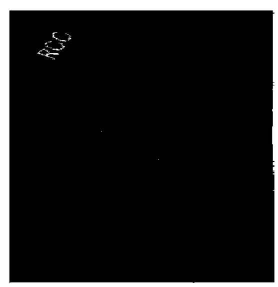
Original digital X-ray lungs image



Contrast enhanced lungs image



Original digital mammography image for breast cancer diagnose



Contrast enhanced mammography image revealing calcium formation

The following mathematical model proposed is useful to any medical imaging application where automatic contrast enhancement and sharpening is needed. Potential areas of impact may include: digital X-ray, digital Mammography, computer tomography scans, nuclear magnetic resonance imagery, telemedicine applications.

The most important performances of this mathematical model are: *dynamic range compression,* i.e. the ability to represent large input dynamic range into relatively small output dynamic range; *sharpening,* i.e. compensation for the blurring introduced into the image by the image formation process. This allows fine details to be seen more easily than before. *color constancy,* i.e. the ability to remove the effects of the illumination from the subject. This allows consistency of output as illumination changes.

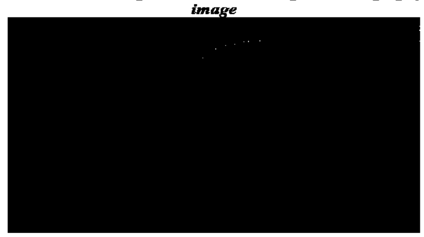
Original digital mammography image for breast cancer diagnose

Contrast enhanced mammography image revealing fibrillar formation

Original computer tomography scan

Contrast and sharpness enhanced computer tomography image

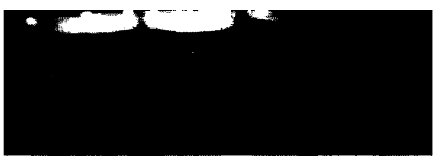
Original digital eye image



Contrast and sharpness enhanced digital eye image

A medical image is represented as a function defined on a two-dimensional spatial domain. Medical images can be classified by the space in which these functions take their values in two groups: scalar images and vector images. Scalar images are defined as functions with real (bounded) values. The most frequent case in practice is when the value at a point (x,y) is the measure of the luminosity at that point. Vector images are defined by real (bounded) vector functions.

Medical color images are the most common particular case of vector images: R(red), G(green) and B(blue) components are considered vector real components. Scalar image values are called gray levels and the functions that define scalar images are called gray level functions. In order to use these gray levels as some algebra elements, the most frequently mathematical model used is the classical one, based on the real numbers algebra (the case of linear processing [1], [2]). This leads to an implicit acceptance of the set of the gray level values as the whole real axis. The result is in contradiction with the fact that gray level functions are bounded. The practical solution to overcome this situation consists in truncating the values as soon as they go out of the true gray level set, be this truncation at the end of chain of operations or at each step. The problem is that in such a way we do not control in the right way the effect of the truncations. Within the mathematical model developed in this paper, the set of gray levels will be the bounded interval (-1,1) put in one-to-one correspondence with the physical interval of e.g. luminosities *(0,M)*, by a linear transformation *y=2(x-M/2)/M.* The problem is to organize the set (-1,1) as a real vector space i.e. closed with respect to an addition (inner group operation) and to a scalar multiplication with real numbers.



Original digital color image of a mouth infection

Contrast and sharpness enhanced digital mouth color image

Original digital color image of eye retina

Contrast and sharpness enhanced color image while preserving color constancy

The key of this approach is to use an adequate isomorphism between $\mathbb{R}$ and (-1,1), respectively between $\mathbb{R}$ and *(0,M)* and the solution is of a logarithmic nature.

The paper is organized as follows: Section 2 and 3 introduces the addition and real scalar multiplication, the scalar product and the norm for the gray levels space respectively for the gray level images space. Section 4 and 5 introduces the addition and real scalar multiplication, the scalar product and the norm for the color space respectively for the color images space. Section 6 presents some experimental results and Section 7 outlines the conclusions.

2. THE REAL VECTOR SPACE OF GRAY LEVELS

We consider as the space of gray levels, the set E= (-1,1). In the set of gray levels E we define the addition $\langle + \rangle$ and the real scalar multiplication $\langle \times \rangle$.
Addition. $\forall u, v \in E, u\langle + \rangle v = \frac{u+v}{1+\frac{u \cdot v}{M^2}}$, where the operations in the right-hand side are meant in $\mathbb{R}$. The neutral element for addition is θ=0. Each element $v \in E$ has as its opposite the element $v\langle + \rangle w = -v$ and this verifies the following equation: $v\langle + \rangle w = \theta$. The addition $\langle + \rangle$ is stable,associative, commutative, has a neutral element and each element has an opposite. It means that this operation establishes on E a commutative group structure. We can also define the subtraction operation $\langle - \rangle$ by $\forall u, v \in E, u\langle - \rangle v = \frac{u-v}{1-\frac{u \cdot v}{M^2}}$. Using subtraction $\langle - \rangle$, we denote the opposite of v by $\langle - \rangle v$.
Scalar multiplication. For $\forall$ $\lambda \in \mathbb{R}$, $\forall u \in E$, we define the product between λ and u by $\forall \lambda \in \mathbb{R}, \forall u \in E, \lambda\langle \times \rangle u = M \cdot \frac{(M+u)^\lambda - (M-u)^\lambda}{(M+u)^\lambda + (M-u)^\lambda}$, where again the operations in the right-hand side of the equality are meant in $\mathbb{R}$. The two operations, addition $\langle + \rangle$ and scalar multiplication $\langle \times \rangle$ establish on E a real vector space structure.
The Euclidean space of the gray levels. We define the scalar product $(.|.)_E$:$E \times E \rightarrow \mathbb{R}$ by $\forall u, v \in E, (u|v)_E = \varphi(u)\varphi(v)$ where φ : (-1,1) $\rightarrow \mathbb{R}$ and φ (x) = arcth(x).With the scalar product $(.|.)_E$ the gray levels space becomes an Euclidean space. The norm $||.||_E$:$E \rightarrow [0,\infty)$ is defined via the scalar product $\forall v \in E, ||v||_E = ((v|v)_E)^{1/2} = |\varphi(v)|$.

3. THE VECTOR SPACE OF THE GRAY LEVEL IMAGES

A gray level image is a function defined on a two-dimensional compact D from $\mathbb{R}^2$ taking the values in the gray level space E. We denote by $F(D, E)$ the set of gray level images defined on D. We can extend the operations and the functions from gray level space E to gray level images $F(D, E)$ in a very natural way:
Addition. $\forall f_1, f_2 \in F(D, E), \forall (x, y) \in D, (f_1\langle + \rangle f_2)(x, y) = f_1(x, y)\langle + \rangle f_2(x, y)$
The neutral element is the identically null function. The addition $\langle + \rangle$ is stable, associative, commutative, has a neutral element and each element has an opposite. As a conclusion, on the set $F(D, E)$ this operation establishes a

commutative group structure.
Scalar multiplication. $\forall\lambda \in \mathbb{R}, \forall f \in F(D,E), (x,y) \in D, (\lambda\langle\times\rangle f)(x,y) = \lambda\langle\times\rangle f(x,y)$ The two operations, addition $\langle+\rangle$ and scalar multiplication $\langle\times\rangle$, establish on $F(D,E)$ a real vector space structure.
The Hilbert space of the gray level images. Let f_1 and f_2 be two integrable functions from $F(D,E)$. We define the scalar product by
$\forall f_1, f_2 \in F(D,E), (f_1|f_2)_{L^2(E)} = \int_D (f_1(x,y)|f_2(x,y))_E dxdy$ With the scalar product the gray level images space $F(D,E)$ becomes a Hilbert space. Further on, we define the norm $\forall f \in F(D,E), ||f||_{L^2(E)}(x,y) = (\int_D ||f(x,y)||_E^2 dxdy)^{1/2}$

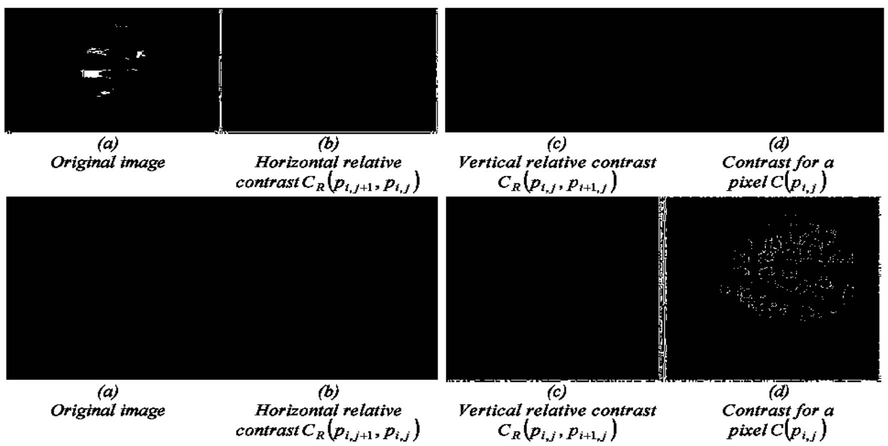
4. THE REAL VECTOR SPACE OF THE COLORS

Next we consider as the space of colors, the cube $E_3(-1{,}1)^3$. Denote by r, g and b (red, green, blue) the three components of a vector $v\in E_3$. In the cube E_3 we define the addition $\langle+\rangle$ and the real scalar multiplication $\langle\times\rangle$.
Addition. $\forall v_1, v_2 \in E_3$ with $v_1 = (r_1, g_1, b_1)$, $v_2 = (r_2, g_2, b_2)$, the sum $v_1\langle+\rangle v_2$ is defined by: $v_1\langle+\rangle v_2 = (r_1\langle+\rangle r_2, g_1\langle+\rangle g_2, b_1\langle+\rangle b_2)$ The neutral element for addition is $\theta = (0{,}0{,}0)$. Each element $v = (r,g,b) \in E_3$ has its opposite element $w=(-r,-g,-b)$ and obviously $v\langle+\rangle w = \theta$. The addition $\langle+\rangle$ is stable, associative, commutative, has a neutral element and each element has an opposite. It follows that this operation establishes on E_3 a commutative group structure. We can also define the subtraction operation $\langle-\rangle$ by $v_1\langle-\rangle v_2 = (r_1\langle-\rangle r_2, g_1\langle-\rangle g_2, b_1\langle-\rangle b_2)$. Using subtraction operation $\langle-\rangle$, we denote the opposite of v, with $\langle-\rangle v$.
Scalar multiplication. For $\forall\ \lambda \in \mathbb{R}$, $\forall\ v = (r,\ g,\ b) \in E_3$ we define the product of λ by v by $\lambda\langle\mathrm{x}\rangle v = (\lambda\langle\mathrm{x}\rangle r, \lambda\langle\mathrm{x}\rangle g, \lambda\langle\mathrm{x}\rangle b)$. The two operations, addition $\langle+\rangle$ and scalar multiplication $\langle\times\rangle$ establish on E_3 a real vector space structure.
The Euclidean space of the colors. We define the scalar product $(.\ |\ .\ )_E : E_3 \times E_3 \to \mathbb{R}$ by $\forall v_1, v_2 \in E_3$ with $v_1 = (r_1, g_1, b_1), v_2 = (r_2, g_2, b_2)$, $(v_1|v_2)_{E_3} = \varphi(r_1)\cdot\varphi(r_2) + \varphi(g_1)\cdot\varphi(g_2) + \varphi(b_1)\cdot\varphi(b_2)$, where φ: $(-1,1) \to \mathbb{R}$ and $\varphi\ (x) = \mathrm{arcth}(x)$. With the scalar product $(.\ |\ .\ )_{E3}$ the color space becomes a three-dimensional Euclidean space. We define the norm $||.||_{E_3} : E_3 \to [0,\infty)$ by $\forall v = (r,g,b) \in E_3 \quad ||v||_{E_3} = (\varphi^2(r) + \varphi^2(g) + \varphi^2(b))^{1/2}$.

5. THE VECTOR SPACE OF THE COLOR IMAGES

A color image is a function defined on a two-dimensional compact D from $\mathbb{R}^2$ taking the values in the colors space E_3. We note by $F(D,E_3)$ the set of color images defined on D. We extend the operations and the functions from colors space E_3 to color images $F(D,E_3)$ in a natural way as follows
Addition. $\forall f_1, f_2 \in F(D,E_3), \forall(x,y) \in D, (f_1\langle+\rangle f_2)(x,y) = f_1(x,y)\langle+\rangle f_2(x,y)$
The neutral element is the identically null function. The addition $\langle+\rangle$ is sta-

ble, associative, commutative, has a neutral element and each element has an opposite. As a conclusion, these operations establish on the set $F(D, E_3)$ a commutative group structure.
Scalar multiplication. $\forall \lambda \in \mathbb{R}, \forall f \in F(D, E_3), \forall (x, y) \in D, (\lambda \langle\times\rangle f)(x, y) = \lambda \langle\times\rangle f(x, y)$.
The two operations, addition $\langle + \rangle$ and scalar multiplication $\langle\times\rangle$ establish on $F(D, E_3)$ a real vector space structure.
The Hilbert space of the color images. Let $f_1$and $f_2$be two integrable functions from $F(D, E_3)$. We define the scalar product by
$\forall f_1, f_2 \in F(D, E_3), \quad (f_1|f_2)_{L^2(E_3)} = \int_D (f_1(x, y)|f_2(x, y))_{E_3} dxdy$ With the scalar product the gray level images space $F(D, E)$ becomes a Hilbert space. Further on, we define the norm
$\forall f \in F(D, E_3), \quad ||f||_{L^2(E_3)}(x, y) = (\int_D ||f(x, y)||_E^2 dxdy)^{1/2}$.



6. THE CONTRAST OF GRAY LEVEL IMAGES

The structure of vector space defined on the set $F(D, E)$ allows us to translate the notion of contrast from classical framework. We denote by $p_i = (x_i, y_i)$ the pairs of coordinates that define the spatial position of a pixel in an image.
The relative contrast between two pixels. The relative contrast between two distinct pixels $p_1, p_2 \in D$, for an image $f : D \to E$, is a logarithmic gray level denoted by $C_R(p_1, p_2)$, and defined by the relation $\forall p_1, p_2 \in D, p_1 \neq p_2 \quad C_R(p_1, p_2) = \frac{1}{d(p_1,p_2)} \langle\times\rangle \frac{f(p_1)-f(p_2)}{1-\frac{f(p_1)\cdot f(p_2)}{M^2}}$, where $d(p_1, p_2)$ is the Euclidean distance between p_1 and p_2 in the $\mathbb{R}^{\ 2}$ plane.
The absolute contrast between two pixels. From the relative contrast C_R we define the absolute contrast by the formulas $\forall p_1, p_2 \in D, p_1 \neq p_2 \quad C_A(p_1, p_2) =$

$|C_R(p_1, p_2)| = \frac{1}{d(p_1,p_2)} \langle\times\rangle \frac{|f(p_1)-f(p_2)|}{1-\frac{f(p_1)\cdot f(p_2)}{M^2}}$.

The contrast for a pixel. The contrast for an arbitrary pixel $p \in D$, for an image $f \in F(D, E)$ is a positive logarithmic gray levels image, denoted by $C(p)$, and defined by the mean of absolute contrast between the pixel p and the pixels $(p_i)_{i=1,n}$ that belong to a neighborhood V. Thus we have the formula $\forall p \in D \quad C(p) = \frac{1}{n}\langle\times\rangle(\langle+\rangle_{i=1}^{n} C_A(p, p_i))$. Usually $n = 8$ and the neighborhood V is a window with 3×3 dimensions and having the pixel p in the center.

Experimental results for the contrast. In the next figures we show the contouring of images with the contrast formulae defined above. We define $C(p)$as the contour image for an initial image, which obtained for each pixel using the contrast formulae. In the nest figures we have represented in section (a) the original image, in section (b) the horizontal relative contrast $C_R(p_{ij+1}, p_{ij}$, in section (c) the vertical relative contrast $C_R(p_{ij}, p_{i+1,j}$ and in section (d) the contrast for a pixel $C(p_{ij})$.

7. CONCLUSIONS

In this paper we presented a new mathematical model for gray level images and also for color images. The main idea is to define an algebraical structure on a bounded interval. The above examples show that the operations (addition, scalar multiplication) and the functions (scalar product, norm) ground an important model for images processing. Using this mathematical model we obtain very simple algorithms for images processing, especially for medical purposes.

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