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SOLVING METHOD FOR MULTIPLE OBJECTIVE FUZZY CONSTRAINTS MIXED VARIABLES OPTIMIZATION

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Abstract In the classical problems of mathematical programming the coefficients of the problems are assumed to be exactly known. This assumption is seldom satisfied by the great majority of real-life problems. Taking into account the multiple criteria optimization in the environment of fuzzy constraints and mixed variables, a solving method is developed here.

Keywords: fuzzy constraints, mixed variables, multiple criteria.

1. INTRODUCTION

In the real life the certainty in and the solidity of data accuracy are illusory. The possibility to obtain an optimal solution is also under the influences of some data missing. Uncertainty has to be taken into consideration when real systems are analyzed. The compromise to accept the uncertainty into the mathematical models must be done working with complex systems. In order to build and to solve real life problem it is natural to interpret the information in fuzzy manner. The philosophy of fuzzy sets theory is related to the model of human thinking and making decisions. Concepts of fuzzy sets theory "crowded" into a lot of research fields since 1980, when fuzzy logic has had a great success in the control systems theory. Real advantages of a fuzzy approach to solve optimization problems could be highlighted when a comparison is made with the stochastic methods to deal with imprecision ([4, 9]).

"The best compromise" (or Pareto-optimality, efficiency, non-domination) is the central concept in multiple criteria optimization only because an optimal solution for one objective function is not necessary an optimal solution for others. In the classical theory weak-efficiency, strong-efficiency, proper-efficiency are gradual definitions for the same essence. Starting with classical definitions, the following concepts appear in the fuzzy multiple objective optimization: M-Pareto optimality (related to the membership functions of "equal" goals), alpha-Pareto optimality (related to the multiple objective programming problems associated with each alpha-level set), M-alpha-Pareto optimality (related to the multiple objective programming problems associated with each alpha-level set after that "fuzzy equal" goals were defined) [1, 2].

In [6], Perkgoz et al. focused on multiobjective integer programming problems with random variable coefficients in objective functions and/or constraints. In [11], Sakawa and Kato presented an interactive fuzzy satisfying method for nonlinear integer programming problems through a genetic algorithm. In [10] α-Pareto optimal solutions were determined for the multiobjective integer nonlinear programming problems having fuzzy parameters in the constraints together with the corresponding stability set of the first kind. The paper [5] presented a method useful in solving a special class of large-scale multiobjective integer problems based upon a combination of the decomposition algorithm coupled with the weighting method together with the branch-and-bound method.

Taking into account the multiple criteria optimization in the environment of fuzzy constraints and mixed variables, a solving method is developed here. In what follows a multiple objective fuzzy constraints mixed variables optimization problem (MOFCMVOP) will be taken into account.

2. THE MOFCMVOP MODEL

Consider the multiple objective integer programming problem with fuzzy constraints and mixed variables

$$\text{"}\min\text{"} \left\{ z\left(x\right) = \left(z_1(x), z_2(x), ..., z_p(x)\right) \mid x \in \overline{X} \right\}, \tag{1}$$

where

(i) $\overline{X} = \left\{ x \in R^n \mid Ax \tilde{\leq} b,\ x \in Z^k \times R^r \right\}$ is the feasible set of the problem,

(ii) A is an $m \times n$ constraint matrix, x is an n-dimensional vector of mixed ($k-$integer, $r-$real) decision variable and $b \in R^m$,

(iii) $p \geq 2$,

(iv) $z_i\left(x\right),\ i = \overline{1,p}$ are the objective functions which could be linear, linear fractional or convex functions (in order to make the new method workable).

The term "*min*" used in Problem (1) is for finding all efficient solutions in a minimization sense in terms of the Pareto optimality. A possible way to handle constraints imprecision is to consider the following parametric problem

$$\text{"}\min\text{"} \left\{ z\left(x\right) = \left(z_1(x), z_2(x), ..., z_p(x)\right) \mid x \in X(\theta) \right\} \tag{2}$$

where $X(\theta) = \left\{ x \in R^n \mid Ax \leq b + \theta b',\ x \in Z^k \times R^r \right\}$ is the feasible set of the problem, $\theta \in [0,1]$ and b' is a given perturbation vector [13].

3. THE SOLVING METHOD

In [7] an algorithm to solving multiobjective linear fractional programming problem (MOLFPP) is presented. In [8] the algorithm is used to solve a fuzzy multiple objective integer programming problem. We will use the above algorithm (under the name $MultiObjAlg(\theta)$) to solve Problem (2) meaning the deterministic problem with the feasible set $X(\theta)$. In order to improve the interactivity of the method different values for θ will be considered.

Wang and Horng [14] proposed an approach to perform complete parametric analysis in integer programming by considering all possible candidates of θ. They defined the principal candidates of θ as being that θ which makes $b_i + \theta b_i^{'}$ an integer. We also work with these θ's. In [13], Wang and Liao proposed a heuristic algorithm to analyze the same fuzzy problem. We will use their method to solving multiple objective problem with integer variables and fuzzy constraints.

For a fixed value θ we start defining Problem (3) as Problem (2) without the integrity restriction of variables.

$$\text{"}\min\text{"}\left\{z\left(x\right) = \left(z_1(x), z_2(x), ..., z_p(x)\right) \mid Ax \leq b + \theta b'\right\}. \tag{3}$$

We obtain an efficient solution for Problem (3) using $MultiObjAlg(1)$. It is a solution for Problem (2) if and only if its components are integer numbers. In this case it is also solution for Problem (1) but with minimal degree in fuzzy environment. Consequently, our next goals are to transform the solution into a mixed ($k-$integer, $r-$real) one and also to improve its fuzzy degree. These goals could be attend modifying values x_i in $[x_i]$ or $[x_i]+1$ for $i = \overline{1,k}$ such that the deviation of the perturbation vector decreases using a modified $back(p, r)$ procedure described in [8].

Taking into account the above remarks the solving algorithm could be described as follows.

- Step 1. Define the thresholds $0 = \theta_1 < \theta_2 < ... < \theta_q = 1$ using the principal candidates of parameters θ. Put $q = 1$.

- Step 2. For $j = q$ down to 1

  – Compute values $x^j = (x_1, x_2, ..., x_n)$ using $MultiObjAlg(\theta_j)$.

  – if $x^j \in Z^k \times R^r$ then favorable STOP with x^j the θ_j-fuzzy degree acceptable solution of the problem.

  – otherwise call $back(1, j, y^j)$ and obtain $x^{j'} = (x'_1, x'_2, ..., x'_n)$. Identify j_{min} such that $Ax' < b + \theta_{j_{min}} b'$. Then $x^{j_{min}}$ is the $\theta_{j_{min}}-$fuzzy degree acceptable solution of the problem.

- Step 3. Describe the problem solution as $\left(y^j\right)_{j=\overline{1,q}}$.

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