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ASIMPTOTIC BEHAVIOR IN VORTEX PHENOMENA: SIGNIFICANT EVENTS

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Abstract The vortex phenomena can be studied both at large and small scale, therefore their applications area is very large, including collection, aggregation and fragmentation of different particles. The turbulent mixing is an important feature of far from equilibrium models. A mixing of a flow implies successive stretching and folding phenomena for system particles, the influence of parameters and initial conditions, and also the issue of significant events - such as rare events - and their physical mean. This paper shows that the turbulent mixing is a basic feature of the vortex phenomena. A comparison between two and three dimensional case is performed.

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1. THE MIXING CONCEPT

In laminar-turbulent transition theory a *flow* is represented by the map $x = \Phi_t(X)$, with $X = \Phi_t(t=0)(X)$; we say that X *is mapped in* x after a time t. The flow is of class C^k, i.e. $\Phi_t(X) \longrightarrow x$ is a diffeomorphism of class C^k. Moreover, $0 < J < \infty$, when $J = \det\left(\frac{\partial x_i}{\partial X_j}\right)$, or, equivalently, $J = det(D\Phi_t(X))$. Here $D = \frac{d}{dx}$. The basic measure for the deformation with respect to X, is the *deformation gradient*, $\mathbf{F} = (\nabla_X \Phi_t(\mathbf{X}))^T$, $F_{ij} = \left(\frac{\partial x_i}{\partial X_j}\right)$, or $\mathbf{F} = D\Phi_t(\mathbf{X})$.

The deformation tensor $\mathbf{F}$ and the associated tensors $\mathbf{C}, \mathbf{C}^{-1}$, (with $\mathbf{C} = \mathbf{F}^T\ \mathbf{F}$) represent the basic quantities in the deformation analysis for the infinitesimal elements. The *length deformation* $\lambda = \lim\limits_{|d\mathbf{X}|\to 0} \frac{|d\mathbf{x}|}{|d\mathbf{X}|}$ and *surface deformation* $\eta = \lim\limits_{|d\mathbf{A}|\to 0} \frac{|d\mathbf{a}|}{|d\mathbf{A}|}$, can be also read as $\lambda = (C : MM)^{\frac{1}{2}}$, $\eta = (\det F) \cdot \left(C^{-1} : NN\right)^{\frac{1}{2}}$, where $\mathbf{M} = d\mathbf{X}/\left|d\mathbf{X}\right|, \mathbf{N} = d\mathbf{A}/\left|d\mathbf{A}\right|$.

In this framework the mixing concept implies the *stretching* and *folding* of the material elements. If in an initial location P there is a material filament dX and an area element dA, the specific length and surface deformations are given by $\frac{D(\ln\lambda)}{Dt} = \mathbf{D} : \mathbf{mm}$, $\frac{D(\ln\eta)}{Dt} = \nabla\mathbf{v} - \mathbf{D} : \mathbf{nn}$, where $\mathbf{D}$ is the deformation tensor. We say that the flow $\mathbf{x} = \Phi_t(\mathbf{X})$ has *a good mixing* if the mean values $D(ln\lambda)/Dt$ and $D(ln\eta)/Dt$ are not decreasing to zero, for any initial position P and any initial orientations $\mathbf{M}$ and $\mathbf{N}$. The *deformation efficiency in length*, $e_\lambda = e_\lambda(X, M, t)$ of the material element dX, is defined [3,4] by $e_\lambda = \frac{D(ln\lambda)/Dt}{(\mathbf{D}:\mathbf{D})^{1/2}} \leq 1$, while, in the case of an isochoric flow (the jacobian equal 1), the *deformation efficiency in surface*, $e_\eta = e_\eta(X, N, t)$ of the area element dA, by $e_\eta = \frac{D(ln\eta)/Dt}{(\mathbf{D}:\mathbf{D})^{1/2}} \leq 1$.

2. STATISTICAL FEATURES OF THE 3D MIXING

The analysis of e_λ and e_η for a 3D model revealed [1], [3] interesting features associating with a vortex experiment for an aquatic algae (*Spirulina Platensis*). The mechanism [5] has: a small version (with a 15-20mm diameter), and a large version (100-300mm diameter), corresponding to two categories of processing particles – at small and at large scale. The analytical discrete study of the mathematical model associated to the phenomena has confirmed the experimental study. The 3D model reads: $\dot{x_1} = G \cdot x_2, \dot{x_2} = K \cdot G \cdot x_1, \dot{x_3} = c$, where $-1 < K < 1,\ \ c = \ const.$

It is a generalization of the 2D version used in [4], a widespread model for isochoric flows. The third component (corresponding to axis z) represents the rotation velocity, supposed to be constant. The Cauchy problem $x_1\,(0) = X_1,\ \ x_2\,(0) = X_2,\ \ x_3\,(0) = X_3$ for the model has a solution $x_i = x_i(X_j)$, i,j=1,2,3 [1,3], where x_i represents the state of the system, at the moment t, with respect to the reference state $X_j, j = 1, 2, 3$ (i.e. it represents the state of the aquatic algae after the vortex experiment).

Let us exhibit the results of the analysis of e_λ and e_η of the material filaments, with the vortex conditions imposed. The studied cases, and the events corresponding to different values of the orientations in length (M_1, M_2, M_3) and in surface (N_1, N_2, N_3) are very few. Their statistical interpretation is realized in [1], including the 2D case. By a *rare event* we mean the event of breaking-up the material filaments, with a corresponding mathematical standpoint in the sudden failing of the running program, or failing the required accuracy. The table represents a synthesis of the work.

3. COMPARISON WITH PERIODIC BEHAVIOR

In [2] there was studied the behavior of a periodic flow: the *tendril-whirl flow (TW)*, introduced by Khakhar, Rising and Ottino (1987). It is a discontinuous succession of *extensional flows and twist maps*.

1.Versors	*2. Versors values*	*3. K*	*4.* $2 \cdot \dot{\gamma}$	*5.t*	*6. Remarks*
$2D\,(M_1, M_2)$	$(1,0)$	0,2	0,001		linear
			0,008		linear
			3,0		linear
	$(-1,1)$	0,2	0,008		linear
			0,001	10	sudden growth
$2D\,(N_1, N_2)$	$(1,0)$	0,2	0,008	6	strong discontinuity
		0,5	0,008	2	strong discontinuity
		0,8	0,008		linear
	$(-1,1)$	0,2	0,008	7	rare event
		0,5	0,008		linear
			3,0		linear
$3D\,(M_1, M_2, M_3)$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$	0,2	0,008		linear
			3,0		linear
		0,8	0,008		rare event
			3,0		linear
			$\sqrt{2}$		rare event
	$\left(\frac{1}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}}, 0\right)$	0,2	3,0		linear
			$\sqrt{2}$		rare event
		0,8	0,008		rare event
			3,0		linear
	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$	0,2	3,0		linear
			$\sqrt{2}$		rare event
			$\sqrt{3}$		rare event
$3D\,(N_1, N_2, N_3)$	$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$	0,2	0,01		rare event
	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$	0,2	0,08	8	maxim
			0,8	8	rare event
			$\sqrt{2}$	5	rare event
			$\sqrt{3}$	4	rare event
	$\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$	0,8	$\sqrt{2}, \sqrt{3}$		rare events

In the simplest case all the flows are identical and the period of alternation extensional/rotational is also constant. However, this case is complex enough and can be considered as the point of departure for several generalizations (smooth variation, distribution of time periods etc). The physical motivation for this flow is that locally, a velocity field can be decomposed into extension and rotation. Mathematically, by polar theorem, a local deformation can be decomposed into stretching and rotation [4]. In the simplest case of the TW model, the velocity field over a single period is given by its extensional part

$$v_x = -\varepsilon \cdot x, v_y = \varepsilon \cdot y, \quad 0 < t < T_{ext}$$

and rotational part

$$v_r = 0, v_\theta = -\omega\left(r\right), T_{ext} < t < T_{ext} + T_{rot},$$

where T_{ext} denotes the duration of the extensional component and T_{rot} the duration of rotational component. The function $\omega(r)$ is positive and specifies the rate of rotation. Its form is quite arbitrary and its most important aspect is that $\frac{d\omega(r)}{dr} = 0$ for some r. The model consists of vortices producing whorls which are periodically squeezed by the hyperbolic flow leading to the formation of tendrils, and the process repeats. In [2] the efficiency of mixing was evaluated only for the extensional part of TW flow. The computation is less complex, and *the deformations in length and surface are less complex than for three-dimensional (non periodic) flow.* This is due to the fact that in the 3D case there are very few parameters. At the same *random values* for the unit vectors, $\sqrt{2}$, $\sqrt{3}$ etc [2], in the 2D case there seems to be no rare events, the functions e_λ and e_η being linear, while in the 3D case the turbulent mixing occurs. While for the vortex phenomena we have a favorable context of *random distributed events* (events with relative linear behavior, with linear-negative behavior, mixing phenomena and rare events), for the TW model (the extensional component), only the deformation in surface seems to have a non constant behavior (the function e_η is decreasing) [2]. The parameter T_{ext} can be measured in seconds, minutes or even in larger units, depending on the context. The same is available for the 3D flow [1], where the turbulence occurs at *small values of the time units, being in agreement with experiments.* Therefore a further analysis for larger T_{ext} would be useful.

References

[1] Ionescu, A., *The structural stability of biological oscillators. Analytical contributions.* Ph.D. Thesis Bucharest, 2002 (Romanian).

[2] Ionescu, A., *Some qualitative features of turbulent mixing for far from equilibrium phenomena*, Proc. of the ICTAMI2005, Alba Iulia.

[3] Ionescu, A., *The discrete time analysis of the turbulence in excitable media*, Proc. of CAIM 2004, Pitesti.

[4] Ottino, J. M., *The kinematics of mixing: stretching, chaos and transport*, Cambridge University Press, 1989.

[5] Savulescu, St.N., *A special vortex tube for particle processing flows*, Rev. Roum. Sci. Techn.-Mec. Appl., **43**, 5(1998), 611-615.