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STUDY OF TRANSIENT PROCESS IN THE SONIC CIRCUIT OF HIGHT-PRESSURE PIPES USED IN LINE FUEL INJECTION SYSTEMS FOR DIESEL ENGINES

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Abstract The modern injection equipment produces a polution level of the emisions which complies with the Europen Norms, i.e. a low fuel consumption level as well as a low level of noise of the Diesel engine. These antagonistic characteristics are achived mainly by optimizing the burning process of the fuel in the burning chanber of the Diesel engine. Obtaining a mixture of optimal aer/fuel ratio depends mainly on a adequate spraying of the fuel such as the drops be as small as possible as well as on the directioning of the pulverised fuel jets by the injector sprayer. For the conventional injection systems, the peak pressure is the the most important measure for the quality of forming the mixture in the burning chamber. Electro-hydraulic analogy, as base of the sonic theory developed by the Romanian scientist George Constantinescu, leads to the possibility of modeling hydraulic systems by electric circuits through sonic resistances, capacities and inductivities. Electro-hydraulic modeling of the high-pressure pipe and injector allows evaluating the adapting condition for optimal adaptation of a chain of sonic qvadripols. By considering the sonic injector circuit at injection phase and writing the transfer functions associated with the sonic quadriples, we are able to obtain the global transfer function, in its operational form. By solving the circuit we can obtain sonic potential differences and also sonic current in operational form. The expressions of pressure and deliveries differences in time range are given as a result of using the Laplace transforms. The presented experimental results show the highest above the pressure peak at injector level, its duration, amplitude of the second peak and attenuation in time domain of the pressure signal.

Keywords: sonic theory, electro-mechanical analogy, compressible fluids
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1. ELECTRO-HYDRAULIC MODELING; THE ASSOCIATION OF THE HYDRAULIC PHYSICAL MEASURES TO THE ELECTRICAL PHYSICAL MEASURES; THE GOGU CONSTANTINESCU FORMULAS

Consider the equations of the rapidly varying motions of fluids in pipes under pressure, [1],

$$(S_1)\begin{cases} \frac{\rho}{A}\frac{\partial q}{\partial t} + \frac{\partial p}{\partial x} + R_u\, q = 0 \\ \frac{A}{\rho\, c^2}\frac{\partial p}{\partial t} + \frac{\partial q}{\partial x} = 0 \end{cases} \tag{1}$$

where ρ is the liquid density, A - the current section of a transmission liquid column, q - flow, p - liquid pressure, c - propagating velocity of the perturbation in liquid columns and R_u - unit resistance coefficient in sonic transmissions. Consider the equations of the long electric lines, [1], [4],

$$(S_2)\begin{cases} L_1\, \frac{\partial i}{\partial t} + \frac{\partial u}{\partial x} + R_l\, i = 0, \\ C_l\, \frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} + G_l\, u = 0, \end{cases} \tag{2}$$

where $u = u(x,t)$ is the line voltage at the x distance from origin, $i = i(x,t)$ is the line current at the x distance from origin, L_l - lineic inductivity, C_l - lineic capacity,R_l - lineic resistance and G_l - lineic conductance. Comparing the systems (S1) and (S2) one can make a formal analogy between electrical and the corresponding hydraulical quantities. Systems (S1) and (S2) coincide if $G_l = 0$. In this case it is possible to achieve quantities in the electro-hydraulic analogy. Table 1 synthesizes the results of the formal comparison of the systems (S1) and (S2).

Notice that in the case of hydraulics circuits, the relation $G_l = 0$ corresponds to the existence of a sonic transmission line without loss of liquid.

Electricity physical measures	i	u	L_l	C_l	R_l
Coresponding physical measures from hydraulics	q	p	ρ/A	$A/\rho c^2$	R_u

Table 1.

2. HIGH PRESSURE PIPE

The transmission of the mechanical power from the pump to the injector proceeds at the sonic speed, (i.e. the sound speed in Diesel oil). The transmission media is Diesel oil modelled as a compressible liquid. The link between the pump and the injector is done by a high pressure pipe of length $L^{'}$. The walls of the high pressure pipe are elastic and the Diesel oil has elastic properties too. It follows the existence of a distributed sonic capacities per unit length of a Diesel gas pipe. Denote by C_1 the sonic capacity of the liquid column whose accumulation volume is V and by C_2 the sonic capacity due to the elasticity of the pipe walls. In fig. 1 we represent the specific capacities $C_1/L^{'}$ and $C_2/L^{'}$, in parallel, and the equivalent sonic capacity C_{SL}, corresponding to the unit length.

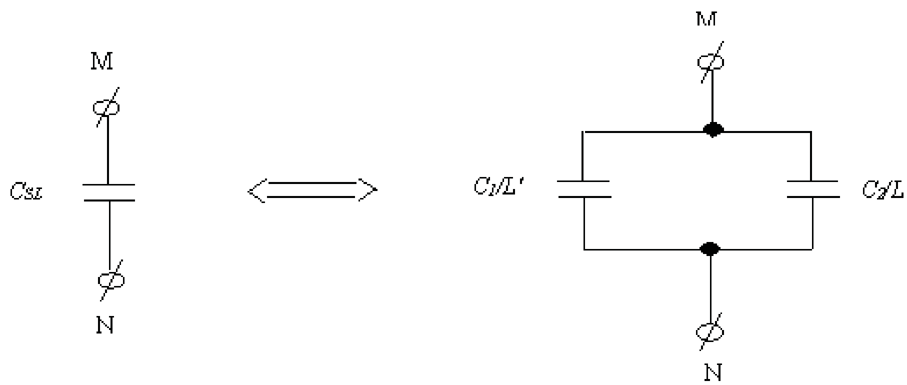

Fig. 1. Distributed sonic capacity.

The sonic capacities have [1],[2] the expressions $C_1 = V/E = L^{'}\Omega/E$, $C_2 = 1.25/E_1 \cdot D_m/e \cdot L^{'}\Omega$, where E is the elasticity module of Diesel oil , $L^{'}$ - the length of the pipe, Ω - the section of the pipe, E_1 - the elasticity module of the material the pipe is made of, D_m - the average diameter of the pipe and e - the thickness of the walls of the pipe. We get $C_{SL} = \frac{C_1+C_2}{L^{'}} = \Omega\ (1/E + 1.25/E_1 \cdot D_m/e)$, where C_{SL} is the distributed sonic capacity per unit length of the high pressure pipe. Sonic capacity C_2 is much (cca 20 - 25 times) smaller than the sonic capacity C_L [2], so that we may consider

$$C_{SL} \approx \Omega/E \tag{3}$$

The inertia of the liquid column determines a distributed sonic inductance per unit length [1], [2]

$$L_{SL} \approx \rho_l/(g\Omega), \tag{4}$$

where is the specific weight of the Diesel oil . Because of the friction between the interior walls of the high pressure pipe and the Diesel oil, we can assume the existence of a sonic resistance [1], [2] distributed per unit length of the liquid column. The expression of the sonic resistance is

$$R_{SL} \approx K^{*}\rho_c/(g\Omega), \tag{5}$$

where K^* is a constant whose value depends on the nature and the speed of the liquid, and ρ_c is the specific mass of the material the pipe is made of.

The high-pressure pipe can be modeled by an infinite cascaded chain of elementary sonic quadruples, with concentrated constants R_{SL}, L_{SL} and C_{SL}. Assuming that the line is homogenous, an elementary quadruple looks like in fig. 2.

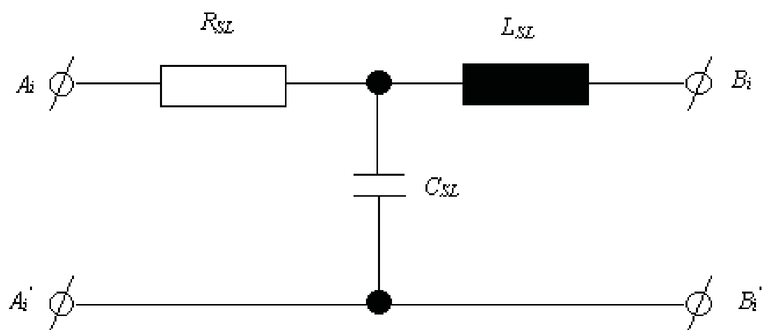

Fig. 2. Electrical equivalent of a sonic circuit associated with an elementary quadruple with concentrated constants R_{SL}, L_{SL} and C_{SL}.

According to the electro-hydraulic modeling, the high pressure pipe can be assimilated with a long electrical line. The electrical equivalent of the sonic circuit associated with the high pressure pipe is represented fig. 3.

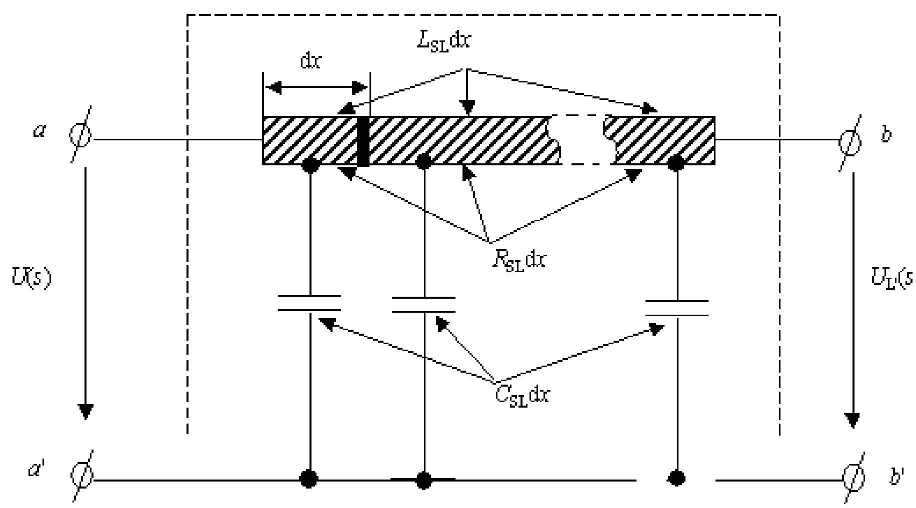

Fig. 3. The electrical equivalent of the sonic circuit associated with the high pressure pipe.

3. THE TRANSFER FUNCTION ASSOCIATED WITH THE SONIC CIRCUIT OF THE HIGH PRESSURE PIPE

Denote by $U(s)$ the sonic input voltage, and by $U_{L'}(s)$ the output sonic voltage of the high pressure pipe, in operational form (fig.3). The transfer function associated with the sonic circuit of the high pressure pipe can be written as $H_{lin}(s) = U_{L'}(s)/U(s)$. The expression of the sonic voltage in operational form in a transversal section of the high pressure pipe at the x distance from the sonic generator has the expression [5], [6]

$$U(x,s) = U(s)\left\{e^{-\gamma x} + \sum_{k=1}^{\infty}(-1)^k \rho_v^k \left[e^{-\gamma(2kL^{'}+x)} - e^{-\gamma(2kL^{'}-x)}\right]\right\}$$

If $x = L^{'}$, where $L^{'}$ is the length of the pipe, we get

$$U_{L'}(s) = U(s)\left\{e^{-\gamma L^{'}} + \sum_{k=1}^{\infty}(-1)^k \rho_v^k(s) \left[e^{-\gamma L^{'}(2k+1)} - e^{-\gamma L^{'}(2k-1)}\right]\right\}$$

and, finally, taking into account the expression of $H_{lin}(s)$,

$$H_{lin}(s) = e^{-\gamma L^{'}} + \sum_{k=1}^{\infty}(-1)^k \rho_v^k(s) \left[e^{-\gamma L^{'}(2k+1)} - e^{-\gamma L^{'}(2k-1)}\right]$$

where γ stands for the propagation constant, and $\rho_v(s)$ is the operational reflexion coefficient. If in the last relation we retain only the first two terms of the series for k = 1, 2, we get

$$H_{lin}(s) \cong e^{-\gamma L^{'}} + \left[\rho_v(s)\left(e^{-\gamma L^{'}} - e^{-3\gamma L^{'}}\right) + \rho_v^2(s)\left(e^{-5\gamma L^{'}} - e^{-3\gamma L^{'}}\right)\right]$$

while if we retain only the first term, for $k = 1$,

$$H_{lin}(s) \cong e^{-\gamma L^{'}} + \rho_v(s)e^{-\gamma L^{'}} - \rho_v(s)e^{-3\gamma L^{'}} \tag{6}$$

4. THE TRANSFER FUNCTION ASSOCIATED WITH THE SONIC CIRCUIT OF THE INJECTOR AT THE INJECTION PHASE

Consider the equivalent of the sonic circuit of the injector initiating the injection (fig. 4). Notice that the equivalent electric circuit of the sonic injector can be achieved exclusively with concentrated sonic elements, R_{ni}, L_i, C_i and R_{di}.

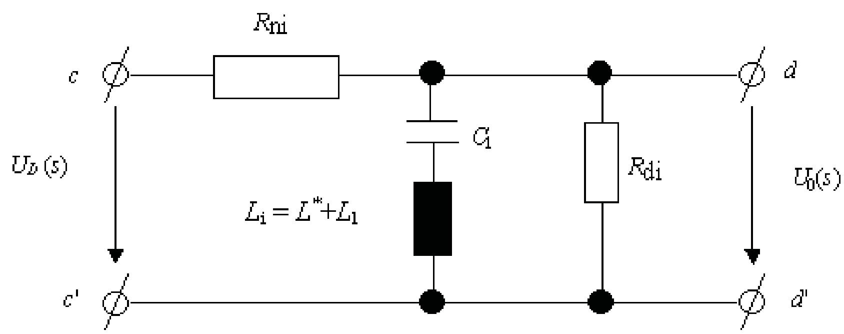

Fig. 4. The electrical equivalent of the sonic circuit of the injector at the initiation of the injection phase.

If the injector is closed, then Diesel gas leaks due to the lack of fitness. These leaks determine the existence of a theoretically infinite sonic resistance, R_{pi}. In this case, the delay line represented by the high pressure pipe ends on an infinite impedance. The needle of the sprayer, the rod and the pressure spring form a sonic circuit $L_i - C_i$ oscillating series. The inertia of the needle and rod determines a sonic inductance L^* and the inertia of the weight of the spring determines a sonic inductance L_1 . The portion between the needle of the injector and the nozzle introduces a sonic resistance, denoted by R_{ni}. The sonic perditance of the nozzle orifices is denoted by S_{di}. These orifices have a constant flow section. The reverse of the sonic perditance Sdi represents the sonic resistance of the nozzle orifices denoted by R_{di}. Denote by $U_{L'}(s)$ the sonic voltage at the output of the high pressure pipe, in operational form, and by $U_0(s)$ the sonic voltage, in operational form, corresponding to the pressure of the Diesel oil at the output of the calibrated orifices of the nozzles of the sonic injector [3]. The operational argument denoted was by s. The transfer function associated with the sonic circuit of the injector has the form

$$H_{inj}(s) = U_0(s)/U_{L'}(s) \tag{7}$$

We determine the expression of the transfer function $H_{inj}(s)$ by transforming the resulting circuit obtained from the electro-hydraulic equivalence of the sonic circuit of the injector. Using the equivalent sonic impedances method, [4], at the first stage, we get the electrical equivalence of the sonic electrical circuit of the injector. This is the first equivalence (fig. 5).

The equivalent sonic impedance Z_1 can be written, in operational form [4] $Z_1(s) = s(L^* + L_1) + (sC_i)^{-1}$. If we consider the sonic impedance Z_{1di}, as equivalent to the sonic impedance Z_1 and R_{di} which are parallel, we obtain the electrical equivalent of the sonic circuit of the injector, i.e. the second equivalence (fig. 6).

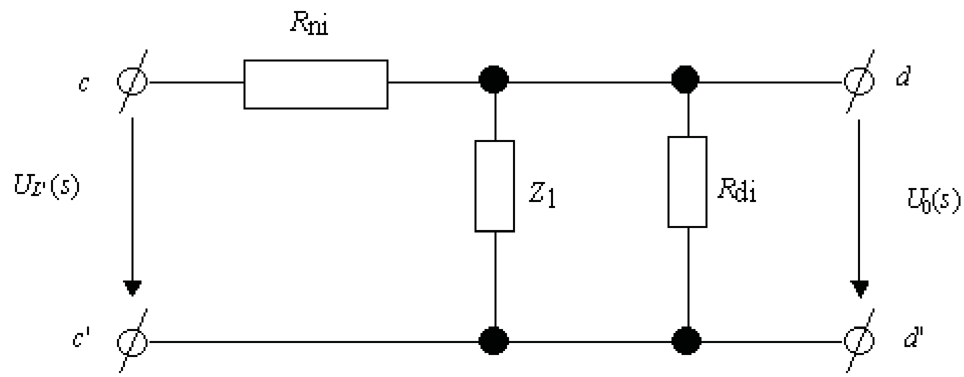

Fig. 5. Electrical equivalent of the sonic circuit of the injector; first equivalence.

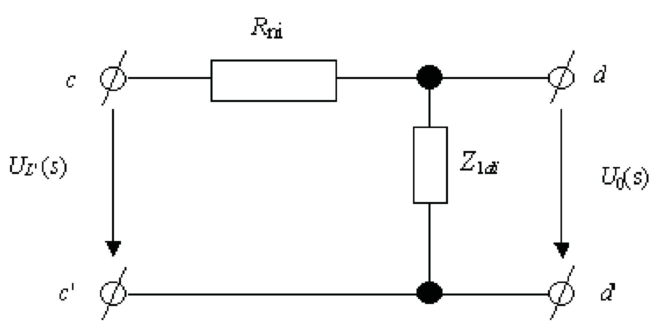

Fig. 6. Electrical equivalent of the sonic circuit of the injector; the second equivalence.

The equivalent sonic impedance Z_{1di} can be written in operational form [4] as $Z_{1di}(s) = Z_1(s)R_{di}/(Z_1(s) + R_{di})$. Using the divisor formula, [4], we get $U_0(s) = Z_{1di}/(R_{ni} + Z_{1di})\ U_{L'}(s)$, whence, by using relation (7), we get the expression of the transfer function associated with the sonic circuit of the injector on the form $H_{inj}(s) = Z_{1di}/(R_{ni} + Z_{1di})$. After successive transformations

and taking into account that $L_i = L^* + L_1$, we get

$$H_{inj}(s) = \frac{T_D s^2 + K_p}{T_1^* s^* + T_2 s + 1} \tag{8}$$

where the following notations have been used: $T_D = R_{di}/(R_{di} + R_{ni})\ C_i L_i = K_p C_i L_i$, $K_p = R_{di}/(R_{di} + R_{ni})$, $T_1^* = C_i L_i$, $T_2 = R_{ni}R_{di}/(R_{di} + R_{ni})\ C_i$. The series circuit $C_i - L_i$ (fig. 4), suggests the apparition of the oscillation phenomenon upon the needle of the sonic injector in the injection phase.

5. MAKING EXPLICIT THE TRANSFER FUNCTION $H_G(S)$ ASSOCIATED WITH THE CHAIN OF SONIC QUADRUPLES PIPE-SONIC INJECTOR

Consider the high pressure pipe and the sonic injector as a quadruples chain connected to the sonic generator in cascade, their interaction being made exclusively on the terminals (fig. 7). The expression of the transfer function $H_g(s)$ can be written [4] as $H_g(s) = H_{lin}(s) \cdot H_{inj}(s)$.

Replacing the expression of $H_{lin}(s)$ and $H_{inj}(s)$ in relations (6) and (8) respectively we obtain, for the injection phase,
$H_g(s) = \left(e^{-\gamma L'} + \rho_v(s)e^{-\gamma L'} - \rho_v(s)e^{-3\gamma L'}\right)\left(\frac{T_D s^2 + K_p}{T_1^* s^2 + T_2 s + 1}\right)$, $H_g(s) = U_0(s)/U(s)$.
Considering the inverse Laplace transforms of the transfer functions $H_{inj}(s)$ and $H_{lin}(s)$ respectively, namely $h_{inj}(t) = L^{-1}\{H_{inj}(s)\}$, $h_{lin}(t) = L^{-1}\{H_{lin}(s)\}$, we may write, [4], $H_g(s) = L\{h_{inj}(t) * h_{lin}(t)\}$ where the convolution product $h_{inj}(t) * h_{lin}(t)$ is given by [4]

$$h_g(t) = h_{inj}(t) * h_{lin}(t) = \int_0^\infty h_{inj}(\tau) h_{lin}(t - \tau) d\tau \ ,$$

where $h_g(t) = L^{-1}\{H_g(s)\}$ and $H_g(s) = L\{h_g(t)\} = \int_0^\infty h_g(t)e^{-st}dt$, s being the operational argument.

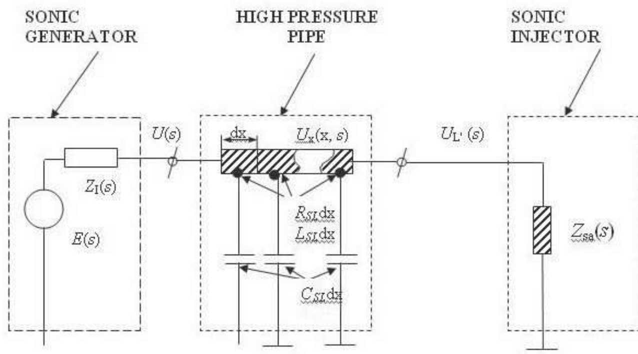

Fig. 7. Electrical equivalent of the sonic circuit associated with a pumping section connected to the injector by a high pressure pipe. Legend: s = operational argument; $E(s)$ = internal sonic voltage of sonic generator, in operational form; $Z_I(s)$ = sonic impedance of sonic generator, in operational form; R_{SL}, L_{SL}, C_{SL} = resistance, inductance, and sonic capacity distributed per unit length of the high pressure pipe; $Z_{sa}(s)$ = equivalent sonic impedance of injector, in operational form; $U(s)$ = the sonic voltage at the input of high pressure pipe, in operational form; $U_{L'}(s)$ = sonic voltage at the output of high pressure pipe, in operational form; $U_x(x,s)$ = sonic voltage in a transversal section of high pressure pipe at the x distance from the sonic generator, in operational form.

6. DETERMINATION OF THE EXPRESSION OF THE SONIC VOLTAGE SIGNAL AT THE INPUT OF THE HIGH PRESSURE PIPE

A pumping section of a Diesel in-line injection pump represents a sonic voltage impulse generator (pressure), (fig. 8). A pumping section (sonic generator) consists of the injection cam, belaying-cleat with reel, the piston of the pumping element, the flow valve and the absorption valve [3], [7]. The high pressure pipe represents a link element of the sonic circuit placed between the absorbing valve and the sonic injector.

Injecting the fuel in the Diesel motor cylinder is a complex phenomenon of transmitting mechanical power from the injection pump to the injector by means of the sonic waves. The transmission media of the sonic waves is the Diesel oil found in the pressure sonic generator, in the high pressure pipe and in the injector. The internal sonic impedance of the sonic generator is written in the operational form as a function of the pressure sonic generator impedances $Z_I^*(s)$, flow valve $Z_{sd}(s)$ and absorption valve $Z_a(s)$, namely

$$Z_I = Z_I^*(s) + Z_{sd}(s) + Z_a(s)$$

According to the divisor formula [4] (fig. 8), the level of the signal at the input of the line is given by the sonic voltage

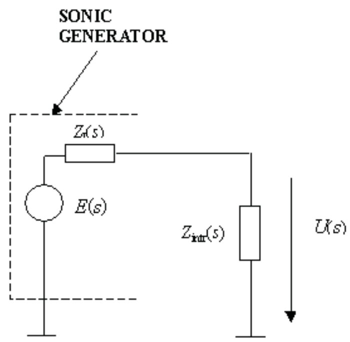

Fig. 8. Electric equivalent of the sonic circuit of a pumping section, connected at a charge of $Z_{intr}(s)$ impedance.

$$U(s) = \frac{Z_{int\,r}(s)}{Z_I^*(s) + Z_{sd}(s) + Z_a + Z_{int\,r}(s)}\; E(s)$$

where $Z_{int\,r}(s)$ is the sonic impedance, in operational form, seen by the sonic generator [4],

$$Z_{int\,r}(s) = \frac{Z_{sa}(s) ch\gamma L' + Z_0(s) sh\gamma L'}{Z_{sa}(s) sh\gamma L' + Z_0(s) ch\gamma L'},$$

$Z_0(s)$ is the operational characteristic sonic impedance of the line (corresponding to the high pressure pipe), and $Z_{sa}(s)$ is the operational sonic impedance of the injector. The operational reflection coefficient, $\rho_v(s)$ has the expression, [4], $\rho_v(s) = [Z_{sa}(s) - Z_0(s)] / [Z_{sa}(s) + Z_0(s)]$. If we neglect the leaks of Diesel oil, the propagation constant of the delaying line represented by the high pressure pipe is written as $\gamma = \gamma(s) = \sqrt{(R_{SL} + L_{SL}) s C_{SL}}$. Replacing R_{SL}, L_{SL} and C_{SL}, out of the relations (3), (4) and (5) respectively, we get $\gamma = \gamma(s) = \sqrt{(K^* \rho_c + s\rho_l) s/(gE)}$. For a lossless line without leakage ($G_{SL} = 0$ and $R_{SL} = 0$ respectively), the characteristic impedance, in operational form, has the expression

$$Z_0(s) = \sqrt{\frac{R_{SL} + sL_{SL}}{G_{SL} + sC_{SL}}} = \sqrt{\frac{L_{SL}}{C_{SL}}} = \frac{1}{\Omega}\sqrt{\frac{\rho_l E}{g}}\,.$$

Taking into account the electrical equivalent of the sonic circuit of the injector (fig. 5), the impedance of the charge can be written in the form

$$Z_{sa}(s) = R_{ni} + \frac{R_{di}\left(sL_i + \frac{1}{sC_i}\right)}{R_{di} + sL_i + \frac{1}{sC_i}} = R_{ni} + \frac{R_{di}\left(1 + s^2 L_i C_i\right)}{1 + sC_i R_{di} + s^2 C_i L_i}$$

The case of in-line Diesel injection systems is typical for transmitting relatively great forces in a small amount of time, $\Delta t \cong 2$ ms, by means of the Diesel oil in high pressure pipes to a sonic receptor placed at a distance from the sonic generator. As the liquid is elastic and has a finite mass the transmission is not instantaneous but depends on the speed of the sound in the Diesel oil. The frequency of the liquid column from the high pressure pipe is, in general, several times greater than the frequency at which injections take place. The pressure waves have the time to travel the pipe several times in-between two successive injections, being reflected at the sonic injector's end as well as at the coupling end of the pumping section. The relexion coefficient $\rho_v \approx 0$, can have positive or negative values. In order for the line to be adapted to the charge it is necessary that $\rho_v \approx 0$. In this case the sonic receptor (the injector) absorbs almost the entire energy of the direct wave.

7. EXPERIMENTAL RESULTS

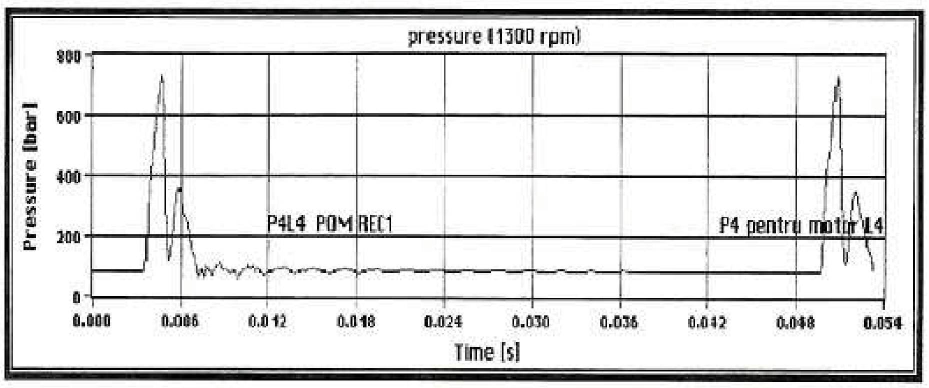

Fig. 9. Pressure diagram at the entry to the high pressure pipe, for the rated power revolution, $n = 1300rpm$.

We report here only two of our experiments. They concern the pressure as a function of time near the coupling where $x = 0$. Namely, the instantaneous pressure is measured at the entry to the high pressure pipe, for the revolution $n = 1300rpm$ corresponding to the rated power (fig. 9), as well as for the revolution $n = 800rpm$, corresponding to the maximum torque (fig. 10). In the other two experiments the pressure was measured at the end close to the injector of the high pressure pipe.

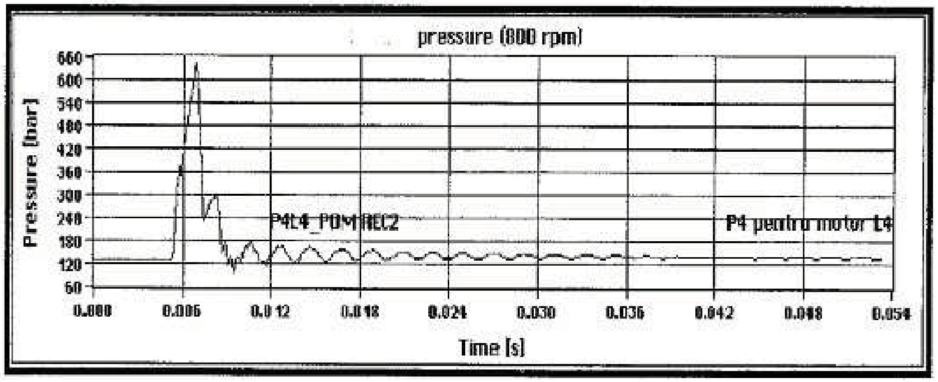

Fig. 10. Pressure diagram at the entry to the high pressure pipe, for the rated power revolution, $n = 800rpm$.

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