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NUMERICAL DISCRETIZATION OF THE ARBITRARY SHAPED REGION BY MEANS OF LAMÉ EQUATIONS

Galina Ribacova

Moldova State University, Chisinau, Republic of Moldova

Abstract The method for creating the regular two-dimensional grids based on equations of longitudinal plate deformation is presented. This problem is solved numerically by means of finite difference method with the posterior using of the iteration process.

The large number of problems connected with numerical modeling of various physical processes leads to necessity of creation of effective methods of discretization of the computational fields with complicated shape. By now, the numerical grid generation became a common tool for use in the numerical solution of partial differential equations on arbitrary shaped regions. Numerically generated grids obviate the difficulties in description of the arbitrary boundary shape from finite-difference method. With such grids all numerical algorithms (including the finite-difference) are implemented on a square or rectangular computational region regardless of the shape and configuration of the initial physical region. Often, in order to solve this problem, methods based on the application of the elliptical partial differential equations are used to describe the interconnection between the computational (ξ, η) and physical (x, y) regions [1]. In the present article the method of creating regular two dimensional curvilinear grids based on the solution of the problem of longitudinal elastic plate deformation is presented. In this problem a system of partial differential equations of elliptic type, namely – Lamé's equations [2] occur.

In order to formulate the problem let us consider the rectangular elastic plate. Consider the rectangular uniform grid with the grid points $(x_i, y_j)\,, x_i = ih_x, y_j = jh_y, i = \overline{0,n}, j = \overline{0,m,}$, ($h_x = l_1/n, h_y = l_2/m$ – the steps of the grid over the corresponding variable, l_1 and l_2 – the dimensions of rectangular plate) marked on this plate. If the plate is subject to longitudinal deformation such that its boundaries take given form (the form of boundaries of the region where the grid must be constructed), then the grid, which was marked on the plate, will be deformed too. As a result of such deformation we obtain the unknown grid. The displacements u and v of the plate points by coordinates x and y respectively satisfy the following system of equations [2]

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 v}{\partial x \partial y} = 0,$$
$$\frac{\partial^2 v}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 u}{\partial x \partial y} = 0, \qquad (1)$$

where μ is Poisson's ratio, the choice of which has an influence upon the grid lines.

The equations (1) can be solved numerically by means of finite difference method on the rectangular grid that has been introduced above. To this aim the equations (1) must be completed with boundary conditions, i.e. the shape of boundaries of initial region is to be known. Then the displacements of boundary points are given, i.e. the following values are known

$$u\,(0,y)\,,\ v\,(0,y)\,,\ u\,(l_1,y)\,,\ v\,(l_1,y)\,,\ y \in [0,l_2]\,,$$
$$u\,(x,0)\,,\ v\,(x,0)\,,\ u\,(x,l_2)\,,\ v\,(x,l_2)\,,\ x \in [0,l_1]\,.$$

Denote by $u_{ij} = u(x_i, y_j)$and $v_{ij} = v(x_i, y_j)$ the values of the unknown functions at the grid points. Then the finite difference approximation of equations (1) is the following

$$u_{ij,x\bar{x}} + \frac{1-\mu}{2} u_{ij,y\bar{y}} + \frac{1+\mu}{4}\left(v_{ij,\overleftarrow{y}x} + v_{ij,y\overleftarrow{x}}\right) = 0,$$
$$v_{ij,y\overleftarrow{y}} + \frac{1-\mu}{2} v_{ij,x\overleftarrow{x}} + \frac{1+\mu}{4}\left(u_{ij,\overleftarrow{y}x} + u_{ij,y\overleftarrow{x}}\right) = 0, \qquad (2)$$

where $i = \overleftarrow{1, n-1}, j = \overleftarrow{1, m-1}$(we here use the generally accepted designation for finite difference derivatives [3]). The created finite difference scheme (2) approximates the initial differential problem (1) with second order relative to h_x and h_y and represents the system of linear algebraic equations of dimensions $2 \times (n-1) \times (m-1)$. The values $v_{0j}, u_{0j}, v_{nj}, u_{nj}, j = \overleftarrow{0, m}$ and $v_{i0}, u_{i0}, v_{im}, u_{im}, i = \overleftarrow{0, n}$ are determined from boundary conditions. Taking into account the large dimensions of the system, its solution must be found by means of the iterative method [3].

The developed algorithm is easy to realize and can be applied to discretize regions with complicated geometrical structures.

References

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