ROMAI J., **3**, 1(2007), 57–64

ON EXTENDED LEWIS CONFORMAL MAPPING IN HYDRODYNAMICS

Camelia Ciobanu, Marian Caţă

"Mircea cel Bătrân" Naval Academy, Constanţa

cciobanu@anmb.ro, mcata@anmb.ro

Abstract The ship hull forms have been described by the well-known classic Lewis transformation [9], and by an extended-Lewis transformation with three parameters, as given by Athanassoulis and Loukakis [1], with practical applicability for any types of ships. We already have presented [2], an algorithmic method solving directly the problems that appear in naval architecture domain concerning the contour of ship's cross-section. In this paper we present how we may extend the Lewis transformation to obtain the contour of the ship's cross section of different types of ships.

1. INTRODUCTION

It was in 1949 that Ursell published his potential theory for determining the hydrodynamic coefficients of semicircular cross sections, oscillating in deep water in the frequency domain. Using this, for the first time a rough estimation could be made of the motions of a ship in regular waves at zero forward speed.

Shortly after that Tasai, Grim, Gerritsma and many other scientists used various already existing conformal mapping techniques (to transform ship-like cross section to a semicircle) together with Urssel's theory, in such a way that the motion in regular waves of more realistic hull forms could be calculated too. Most popular was the 2-parameter Lewis conformal mapping technique.

It is necessary to approximate the ship's shape by continuous functions, in order to get some practical results. A method, which has imposed itself during

the last few years, is that of multi-parameter conformal mapping, with good results also in the case of extreme bulbous forms.

The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrary shape of a cross section in a complex plane can be derived from the more convenient circular section in another complex plane. In this manner, hydrodynamic problems can be solved directly by using the coefficients of the mapping function.

The general transformation formula is given by:

$$f(Z) = \mu_s \sum_{k=0}^{n} a_{2k-1} Z^{-2k+1}, \tag{1}$$

with $f(Z) = z, z = x + iy$ is the plane of the ship's cross section, $Z = ie^{\alpha}e^{-i\varphi}$ is the plane of the unit circle, $\mu_s = 1$, a_{2k-1} are the conformal mapping coefficients $(k = 1, ...n)$, n is the number of parameters.

Therefore we can write

$$x + iy = \mu_s \sum_{k=0}^{n} a_{2k-1} (ie^{\alpha}e^{-i\varphi})^{-(2k-1)}, \tag{2}$$

$$x + iy = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} [icos(2k-1)\varphi - sin(2k-1)\varphi]. \tag{3}$$

From the relation between the coordinates in the z - plane (the ship's cross section) and the variables in the Z - plane (the circular cross section), it follows

$$x = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} sin(2k-1)\varphi, \tag{4}$$

$$y = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} cos(2k-1)\varphi. \tag{5}$$

Now by using conformal mapping approximations, the contour of the ship's cross section, follows from putting $\alpha = 0$ in (4) and (5). We get

$$x_o = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} sin(2k-1)\varphi,$$

$$y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} cos(2k-1)\varphi.$$

The breadth on the waterline of the approximate ship's cross section is defined by

$$B_0 = 2\mu_s \beta, \text{ with } \beta = \sum_{k=0}^{n} a_{2k-1},$$

and the draft is defined by

$$D_0 = 2\mu(s)\delta, \text{ with } \delta = \sum_{k=0}^{n} (-1)^k a_{2k-1}.$$

The breadth on the waterline is obtained for $\varphi = \pi/2$, that means

$$x_{\pi/2} = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} sin(2k-1)\pi/2,$$

hence

$$x_{\pi/2} = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1}, \text{ and } B_0 = 2x_{\pi/2}.$$

The scale factor is $\mu_s = B_o/2\beta$ and the draft is obtained for $\varphi = 0$:

$$y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} cos(2k-1)0, \text{ hence } y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \text{ and } D_0 = y_0$$

with $\mu_s = D_0/\delta$.

2. EXTENDED LEWIS CONFORMAL MAPPING

We can obtain better approximations of the cross sectional hull form by taking into account also the first order moments of half the cross section about the x-and y-axes. These two additions to the Lewis formulation were proposed by Reed and Nowacki [11] and have been simplified by Athanassoulis and Loukakis [1] by taking into account the vertical position of the centroid of the cross section. This has been done by extending the Lewis transformation from n=2 to n=3 in the general transformation formula.

The three-parameter extended Lewis transformation of a cross section is defined by

$$z = f(Z) = \mu_s a_{-1} Z + \mu_s a_1 Z^{-1} + \mu_s a_3 Z^{-3} + \mu_s a_5 Z^{-5}, \tag{6}$$

where $a_{-1} = 1$, μ_s is the scale factor and the conformal mapping coefficients a_1, a_3, a_5 are called Lewis coefficients. Then, for $z = x + iy$ and $Z = ie^{\alpha}e^{-i\varphi}$, that is $Z = ie^{\alpha}[cos(-\varphi) + isin(-\varphi]$, we have

$$x = \mu_s(e^{\alpha} sin\varphi + a_1 e^{-\alpha} sin\varphi - a_3 e^{-3\alpha} sin3\varphi + a_5 e^{-5\alpha} sin5\varphi)$$

and

$$y = \mu_s(e^{\alpha} cos\varphi - a_1 e^{-\alpha} cos\varphi + a_3 e^{-3\alpha} cos3\varphi - a_5 e^{-5\alpha} cos5\varphi).$$

For $\alpha = 0$ we obtain the contour of the so- called extended Lewis form

$$x_0 = \mu_s(sin\varphi + a_1 sin\varphi - a_3 sin3\varphi + a_5 sin5\varphi),$$

and

$$y_0 = \mu_s(cos\varphi - a_1 cos\varphi + a_3 cos3\varphi - a_5 cos5\varphi)$$

where the scale factor μ_s is

$$\mu_s = B_s/2(1 + a_1 + a_3 + a_5) \text{ or } \mu_s = D_s/(1 - a_1 + a_3 - a_5),$$

in which B_s is the sectional breadth on the waterline and D_s is the sectional draught. The half breadth to draft ratio H_0 is given by

$$H_0 = \frac{B_s}{2D_s} = (1 + a_1 + a_3 + a_5)/(1 - a_1 + a_3 - a_5).$$

An integration of the extended Lewis form delivers the sectional area coefficient:

$$\sigma_s = A_s/B_s D_s = \pi/4 \cdot (1 - a_1^2 - 3a_3^2 - 5a_5^2)/[(1 + a_3)^2 - (a_1 + a_5)^2]$$

in which A_s is the area of the cross section, $A_s = \pi/2 \cdot \mu_s^2(1 - a_1^2 - 3a_3^2 - 5a_5^2)$ and $B_s D_s = 2[(1 + a_3)^2 - (a_1 + a_5)^2]$.

Now the coefficients a_1, a_3, a_5 and the scale factor μ_s will be determined in such a manner that the sectional breadth, the draft and the area of the approximate cross section and of the actual cross section are identical. We have already presented [2] a typical and realistic form. More precisely we have considered a dry bulk carrier of 55.000 tone deadweight capacity and "Mircea" school ship. That application was made in Java language and created both a text file and a graphical chart. A more complex expression has been obtained by Athanassoulis and Loukakis [1] for the relative distance of the centroid to the keel point

$$k = \frac{KB}{D_s} = 1 - \frac{\sum_{i=0}^{3}\sum_{j=0}^{3}\sum_{k=0}^{3} A_{ijk} a_{2i-1} a_{2j-1} a_{2k-1}}{H_0 \sigma_s \sum_{i=0}^{3} a_3^{2i-1}}$$

in which

$$A_{ijk} = \frac{1}{4}\left\{\frac{1-2k}{3-2(i+j+k)} - \frac{1-2k}{1-2(i-j+k)} + \frac{1-2k}{1-(i+j-k)} + \frac{1-2k}{1-2(-i+j+k)}\right\}.$$

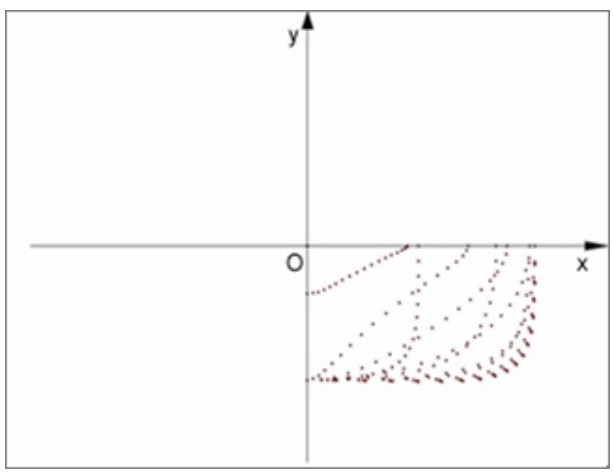

Fig. 1. The contour of the ship's cross section of the dry bulk carrier.

Figs. 1 and 2 were obtained from the previous method with a software package specially developed for this purpose [2].

The graphical representation of the points shows the contour of the ship's cross section of the dry bulk carrier and "Mircea" school ship.

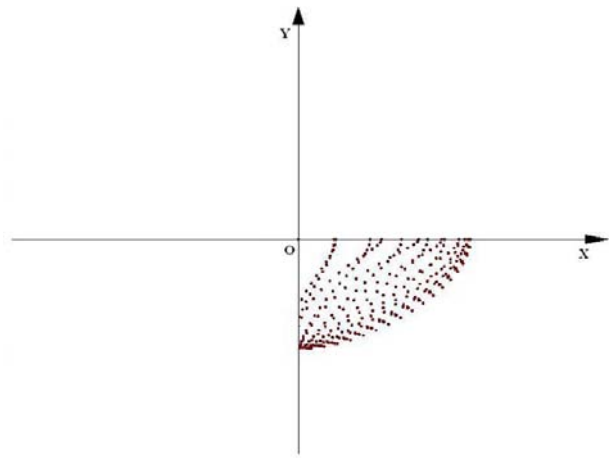

Fig. 2. The contour of the ship's cross section of the "Mircea" school ship.

3. CONCLUSIONS

This is a mathematical solution in order to obtain the contour of the ship's cross section of different types of ships, using conformal mapping approximations.

The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrary shape of a cross section in a complex plane can be derived from the more convenient circular section in another complex plane.

In this manner hydrodynamic problems can be solved directly with the coefficients of the mapping function.

In the future we hope to obtain much better graphical representation by considering three or more coefficients.

Acknowledgements

This paper was supported by the Grant 106/17.08.2007 of Romanian Academy.

References

[1] G. A. Athanassoulis, T. A. Loukakis, *An extended-Lewis form family of ship sections and its applications to seakeeping calculations*, International Shipbuilding Progress, **32**, 366 (1985).

[2] C. Ciobanu, M. Cata, A. R. Anghel, *Conformal mapping in hydrodynamics*, Bulletin of the "Transilvania" University of Brasov, **13**, 48 (2006).

[3] Y. Ikeda, Y. Himeno, N. Tanaka, *A prediction method for ship rolling*, Technical Report, Department of Naval Architecture, University of Osaka Prefecture, Japan, 0045, 1978.

[4] B. de Jong, *Computation of hydrodynamic coefficients of oscillating cylinders*, Tehnical Report, Netherlands Ship Research Centre TNO, Shipbuilding Department, Delft, The Netherlands, 145-S, 1973.

[5] J. M. J Journee, *Strip theory algorithms, Revised report,* Tehnical Report, Delft University of Technology, Ship Hydromechanics Laboratory, The Netherlands, 912, 1992.

[6] J. M. J Journee, *Verification and validation of ship motions program SEAWAY*, Delft University of Technology, Ship Hydromechanics Laboratory, The Netherlands, 1213, 2001.

[7] J. M. J Journee, W. W. Massie, *Offshore hydromechanics*, Lecture Notes, Delft University of Technology, Ship Hydromechanics Laboratory, The Netherlands, 2001, Internet: www.shipmotions.nl.

[8] C. V. Kerczeck, E. O. Tuck, *The representation of ship hulls by conformal mapping functions*, Journal of Ship Research, **13**, 4, (1969).

[9] F. M. Lewis, *The inertia of water surrounding a vibrating ship*, Transactions SNAME, 1929.

[10] H. Lamb, *Hydrodynamics*, Dover, New York, 1932.

[11] M. A. Reed, H. Nowacki, *Interactive creation of fair ship lines*, Journal of Ship Research, **18**, 2 (1974).

[12] F. Tasai, *Improvements in the theory of ship motions in longitudinal waves*, Proceedings 12th I.T.T.C, 1969.

[13] F. Ursell, *On the heaving motion of a circular cylinder on the surface of a fluid*, Quaterly Journal of Mechanics and Applied Mathematics, II, 1949.

[14] M. Vantorre, J. M. J. Journee, *Validation of the strip theory code SEAWAY by model tests in very shallow water*, Colloquium on Numerical Modelling, 23-24 October 2003, Antwerp, Belgium, Internet: www.shipmotions.nl.