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LIMITER CONTROL OF CHAOTIC ECONOMIC DYNAMICS

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Abstract By the Hard Limiter Control (HLC) a chaotic regime is turned into a cyclic or a stationary one. The numerical examples are worked out by us, completing the quite poor literature in the field. The MathCad was used.

1. CONTROL STRATEGY

The increase and decrease of modern economy, not only that strongly affects society, but it also has a direct impact on population. Cyclic phenomena and economic crisis are unavoidable in the process of economic evolution. If these could be foretold and thus their origin could be understood, then they could be redirected towards a slower evolution.

Until recently economic cycles had been thought of as undesirable, being regarded as a variation of demand (of companies' investment and of population's consume). Recently, a procedure for the control of macroeconomic development has been elaborated. One of the conclusions was that the control must be maintained constant during a cycle so that the negative effects could be minimized. In order to obtain this effect and to understand the resulted solution the measure of control has been applied to simple economic models.

The problem of economic foretelling is strictly connected to that of chaotic processes and the elaboration of new strategies to avoid these has been attempted. A disadvantage of control is that it is based on the observation of the past. In following a principle that could generate cycles in economic models will be identified. As it will be seen, to obtain a first-period regime, a drastic and permanently maintained control is necessary.

Firstly, the logistic equation will be introduced as a simple economic model which has a general characteristic. Using this model, the source of the best control cycles could be identified. When an exponential increase is possible in economy, the problems are as less as possible. However, in most of the cases, foretelling is difficult.

2. HARD LIMITER CONTROL (HCL)

In our paper we adopt the definition according to which chaos corresponds to in an infinite number of instable periodic orbits with divergent periodicities. In order to use this behaviour supply of the system there have been elaborated many methods of control and stabilization of the orbits. The method which will be described is based on simple limits, i.e. it consists in limiting the phase space. As a result of this procedure, the orbits with points in the forbidden fields are eliminated. It could be noticed that the modified system has a tendency to replace the chaotic behaviour with a periodic one.

The typical model chosen in order to analyze the economic increase is that of the logistic equation. As it clearly follows from the representation of the Feigenbaum diagram (fig. 1) there could be distinguished several specific situations which are obtained by the continuous increase of the parameter a.

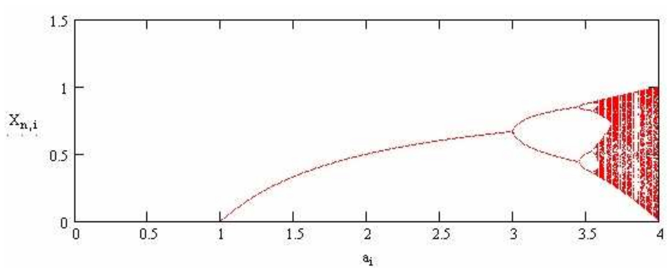


Fig. 1. Feigenbaum diagram.

Case 1. $0 < a < 1$. In this interval, iterative process, i.e. the function (solution) values is stabilized to the attractor which is the point of zero value. Thus this interval could be seen as 'bankruptcy' and, in case of economy, a state of economic collapse.

Case 2. $1 < a < 3$. During iterations the values of the function are stabilized at a certain value determined by the parameter a. This could be interpreted as a period of increase in the analyzed economy (firm) and it characterizes the state of nonnull equilibrium.

Case 3. $3 < a < 3.659$. In this interval there occur second-order bifurcations and the powers of two. This interval will characterize the state of economic cycles (cycle two, four etc.)

Case 4. $3.659 < a < 4$. In this interval there occurs the chaos. There exist many windows in which the function stabilizes itself in cycles of order 3 or multiple of 3. In the natural course of economy, its state will follow in one of the above cases. There is a possibility for the executive (of the company or even of the country if the national economy is involved) to interfere in order to change the existing state state of economy. The measures of intervention are known as hard limiter control (HLC). By introducing the control, the economic regime could be modified by going from one interval of the logistic function to another.

There are two types of HLC: superior limiter; inferior limiter. In the case of superior limiter, during iterations it is necessary that the values of the studied function k_{t+1} should not go over a settled maximum value, called threshold h. In the case of inferior limiter, during iterations it is necessary that the values of the studied function k_{t+1} should not go over a settled minimum value.

In the following we analyze the effects of HLC. While the superior limiter is mentioned in the existing studies, the inferior limiter is hardly mentioned. Actually, the intervention of the executive in the process of economy and thus of the capital k_{t+1}, is accomplished by employing either superior limiter or inferior limiter in both senses. Each mentioned procedure will be analyzed.

The influence of the introduction of the limiter factor will be illustrated in several cases both by graphical and numerical methods. Remark that the analyses that uses only (mathematical relations) or graphical methods, are the most useful, but in many cases they are not sufficient. For instance, we show that if the cycles exist, there are relatively small differences between the values of the iterated function which can be overlooked when the graphical representation is considered.

When the numerical values are also taken into consideration, the analysis is considerably improved. In such cases, the employment of Mathcad is the best. We show that by modifying the value of the control factor h, the regime of the logistic function which describes the economic model could be changed. Thus $h = 1$ means that there was no intervention, because the maximum value of the field is 1 and thus the values of the function do not suffer any limitation. The more the value of h is decreasing, i.e. the limiters are tougher and tougher, so is the control. It will be shown that if we intensify the control factor, what is obtained through limiter factors of low values out of chaotic regime could change into cycles characterized regime (of period 8, 4 or 2) or even into equilibrium regime. Moreover, it could be demonstrated that it could even change into cyclic regime of period 3 or $3 \cdot 2^n$ which is situated in the interior of the chaotic field.

Now we show how the value of the limiter factor modifies the regime of the logistic function. We shall start with the point at which, without limitation, the logistic function is situated in chaotic regime.

3. THE CHANGE OF A CHAOTIC REGIME INTO THE CYCLIC REGIME OF PERIOD 8, 4, 2, OR EQUILIBRIUM

The following cases are studied

1) The regime of period 8 is done for $a = 3.9$ and $h = 0.92$ (fig.2);

2) the regime of period 4, for $a = 4$ and $h = 0.91$, (fig.3);

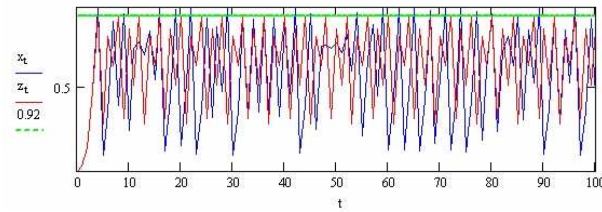


Fig. 2. The change from the chaotic regime into a cyclic regime of period 8 is done for $a = 3.9$ and $h = 0.92$.

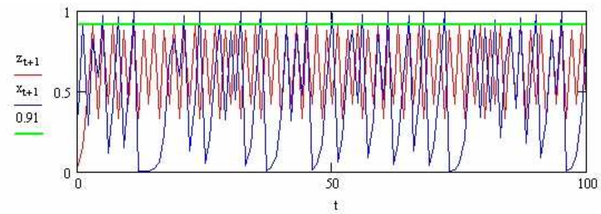


Fig. 3. The change from the chaotic regime into a cyclic regime of period 4 for $a = 4$ and $h = 0.91$.

3) the regime of period 2, for $a = 4$ and $h = 0.9$, (fig.4), while,

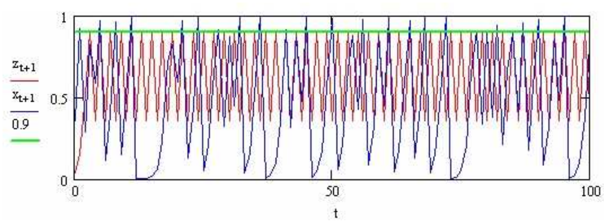


Fig. 4. The change from the chaotic regime into a cyclic regime of period 2 is done for $a = 4$ and $h = 0.9$.

4) in equilibrium regime, for $a = 4$ and $h = 0.75$, (fig.5);

5) the regime of period 10 is done for $a = 4$ and $h = 0.75$, (fig.6).

We shall indicate the initial values of the logistic function which describes the economic model of Day and the relations of programming in MathCAD. The initial logistic function (without limiter) is x_{i+1}, whereas z_{i+1} is the lo-

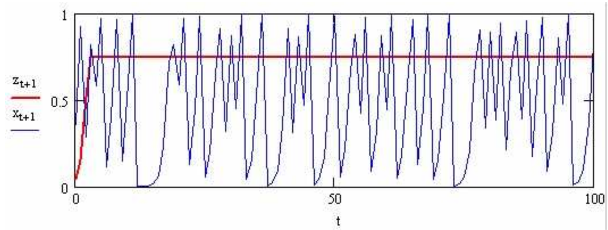


Fig. 5. The change from the chaotic regime into an equilibrium regime, for $a = 4$ and $h = 0.75$.

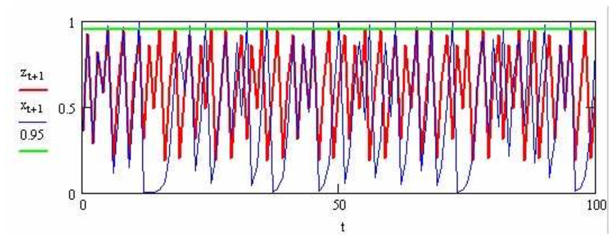


Fig. 6. The change from the chaotic regime in a cyclic regime of period 10, for $a = 4$ and $h = 0.95$.

gistic function in which a limiter h was introduced. The results of the computations are represented in figs. 2-6.

4. THE EMPLOYMENT OF HARD LIMITER CONTROL IN ORDER TO MODIFY THE CYCLIC REGIMES

The employment of Hard Limiter Control is useful when the chaotic regime is changed into the cyclic regime or equilibrium but also in other cases. As it will follow, through the employment of Hard Limiter Control cyclic regimes could be changed too. Thus, an initial stage in cyclic regime of period 8 could be turned into a cyclic regime of smaller periods: 4 or 2 or equilibrium. We are going to illustrate a few situations:

- the change from cyclic regime of period 8 into a cyclic regime of period 4, for a = 3.56 and h = 0.87 (fig. 7);

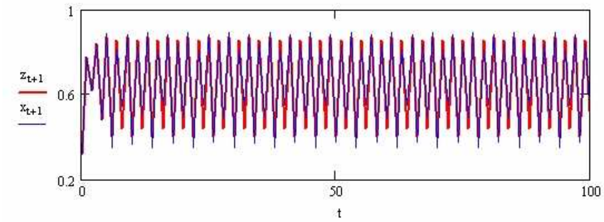


Fig. 7. The change from the cyclic regime of period 8 into a cyclic regime of period 4, for $a = 3.56$ and $h = 0.87$.

- the change from cyclic regime of period 8 into a cyclic regime of period 2, for a) a = 3.56 and h = 0.86; b) a = 3.56 and h = 0.75;

- the change from cyclic regime of period 8 into equilibrium regime, for $a = 3.56$ and $h = 0.7$.

5. THE CHANGE FROM THE CHAOTIC REGIME INTO A CYCLIC REGIME OF PERIOD 3 OR $3 \cdot 2^N$

Previously we mentioned the existence of cycles of period 3 or $3 \cdot 2^n$ in the windows which are situated in the interval characterized by chaos. We shall see that, similarly, by introducing an inferior or superior limiter control, the logistic function could be modified so that the initial chaotic regime could be changed into cyclic regime of period 3 or $3 \cdot 2^n$. We shall illustrate two such situations:

- the change from the chaotic regime into a cyclic regime of period 6, for $a = 4$ and the superior limiter $h = 0.97$ (fig. 8);

- the change from the chaotic regime into a cyclic regime of period 12, for $a = 4$ and the superior limiter $h = 0.03$ (fig.9)

6. REMARKS

One of the most important cases in which intervention is needed is the one in which the model of the logistic function is situated in the chaotic regime. It

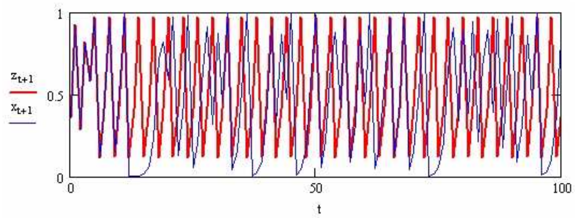


Fig. 8. The change from the chaotic regime into a cyclic regime of period 6, for $a = 4$ and $h = 0.97$.

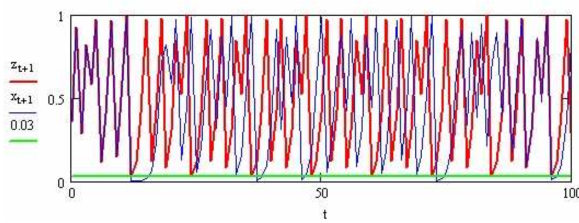


Fig. 9. The change from the chaotic regime into a cyclic regime of period 12 is done for $a = 4$ and $h = 0.03$.

has been shown that by applying different ways of control (determined by the value of the limiter factor h), there could be a change from the chaotic regime into a cyclic one (of period 8, 4 or 2). Moreover, with more stressed values of h, the change could be done even into a stationary state.

Necessities and possibilities of control appear not only when the logistic function is in chaotic regime, but also when it is in a cyclic regime. As it has been shown, in this case, by using limiters, it is possible to change cyclic regimes of higher periods into cyclic regimes of lower periods. For instance, from 8 to 4 and from 4 to 2. In this case it is also possible to change the cyclic regimes into equilibrium.

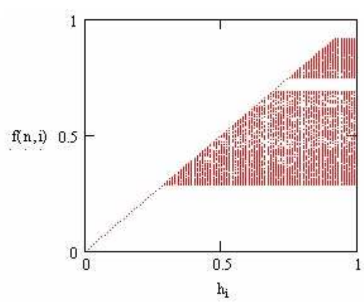
Through less tougher limiters h, values close to 1 (for superior limiters) and close to 0 (for inferior limiters), there could be made the change from the chaotic regime into the cyclic regime of period 3 or $3 \cdot 2^n$, or even other periods. The series of time which follows for the case when $a = 4$ and $h = 0.9$ is illustrated in fig. 4.

Through this process, the previous chaotic dynamics was forced to produce a periodic oscillation of period 2. The average density increases in comparison to the density of the model without limiter after the same number of iterations and the same x_0. Around the value of 0.9 there is a cycle of period 2 whose amplitude decreases with h and even disappears if $h = \frac{A-1}{A} = 0.75$(where the fixed point is reached); the result would be equilibrium, eventually. If $h < \frac{A-1}{A}$, the dynamics is pushed towards the limiter. For a relatively large interval of h the average density is higher than in the case of a model without limiter. This result is counterintuitive because the density has been applied the superior limiter and the expectation would have been a decreasing average. An heuristical explanation of this paradox could be found by means of fig. 2,4. Cobweb algorithm is applied both to the initial logistic equation and to the limiter case. The algorithm starts with the value $x_0 = 0.5$ because at this value, the maximum value of the logistic diagram could be obtained. This allows dynamics to be returned into the descendant branch that intersects the abscissae axis. So, in the chaotic regime, the regime of ergodic orbit fills the whole [0,1] interval. On the other hand, in the case of the limiter model, the changing does not explore decreasing interval because the peak is brought to the value of the limiter factor and thus the explored interval is much smaller. The difference of possible densities is underlined with a thick line. It could be easily noticed that the missing inferior interval is higher than the missing superior interval. This means that the interval has been pushed up, which explains the higher values of the density. The flat superior limiter explains why the dynamics in the case of limiter control is forced to be periodically oscillating or even stabilize to the values of the fixed points. Once the orbit has reached the limit region (due to ergodicity, too) image will always be the same. So the system is lead into a very stable cycle. The length of this cycle can be determined (Sinha 1994). If the value of the function $f(x_t)$ from k iteration exceeds h limit, then period k can be obtained. In this way the average density can be calculated.

As it has been remarked, the possibilities of intervention through inferior limiter (at minimum allowed values) have similar effects as those obtained through superior limiter. Fig. 10 points out the asymptotic dynamics of the model in accordance with the superior limiter factor h. We denoted $f(n,i) = \min(aX_{n,i}(1 - X_{n,i}), h_i)$.

7. CONCLUSIONS

As a result of the previous exemplifications there could be drawn important conclusions in reference to the study of economic increase and of possibilities of intervention on behalf of executive departments in order to improve its evolution. The possibilities of intervention consist mostly in measures that lead to limitation of the function growth and k (t) = K (capital) / L (work force) of PIB (in the case of national economy), of course. Limiters refer to the highest or lowest values possible for capital. These could be done by means of superior HLC or inferior HLC.



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