ROMAI J., **3**, 2(2007), 263–265

ON SOME APPROACH TO NUMERICAL GRID GENERATION

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Abstract In the present article the method of creating the regular two dimensional curvilinear grids based on the solution of the problem of longitudinal elastic plate deformation is presented.

A large number of problems connected with numerical modeling of various physical processes leads to necessity of creation of effective methods of discretization of the computational fields possessing complicated shape. The numerical grid generation now became a common tool for use in the numerical solution of partial differential equations on arbitrary shaped regions. Numerically generated grids obviate the difficulties in description of the arbitrary boundary shape from finite difference method. With such grids all numerical algorithms (including the finite difference) are implemented on a square or rectangular computational region regardless of the shape and configuration of the initial physical region. In order to solve this problem there are often used the methods based on the application of the elliptical type partial differential equations to the description of the interconnection between the computational (ξ, η) and physical (x, y) regions.

Further we present the method of building two dimensional curvilinear grids on the basis of the solution of the problem of the longitudinal elastic plate deformation.

In order to formulate the problem consider the rectangular elastic plate. Let the rectangular uniform grid with the grid points $(x_i, y_j)\,, x_i = ih_x, y_j = jh_y, i = \overline{0,n}, j = \overline{0,m},$ ($h_x = l_1/n, h_y = l_2/m$ are the steps of the grid on corresponding variable, l_1 and l_2 are the dimensions of rectangular plate) be marked on this plate. If the plate is subject to longitudinal deformation so as its boundaries take some given form (the form of the boundaries of the region where the grid must be constructed), then the grid, which was marked on the plate, will be deformed too. As a result of this deformation we obtain the unknown grid. The displacements u and v of the plate points by coordinates x and y respectively satisfy the following system of equations [2]

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 v}{\partial x \partial y} = 0,$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 u}{\partial x \partial y} = 0, \tag{1}$$

where μ is the Poisson ratio, the choice of which have an influence upon the grid lines.

The equations (1) can be solved numerically by means of finite difference method on the rectangular grid that has been introduced above. For this purpose the equations (1) must be completed with boundary conditions, i.e. the shape of boundaries of initial region is to be known. The displacements of boundary points are given, i.e. the following values are known

$$u(0,y), v(0,y), u(l_1,y), v(l_1,y), y \in [0,l_2]$$

$$u(x,0), v(x,0), u(x,l_2), v(x,l_2), x \in [0,l_1]$$

Let us denote by $u_{ij} = u(x_i, y_j)$ and $v_{ij} = v(x_i, y_j)$ the values of the unknown functions at the grid points. Then the finite difference approximation of equations (1) is the following

$$u_{ij,x\overline{x}} + \frac{1-\mu}{2}u_{ij,y\overline{y}} + \frac{1+\mu}{4}(v_{ij,\overleftarrow{y}x} + v_{ij,y\overleftarrow{x}}) = 0,$$

$$v_{ij,y\overleftarrow{y}} + \frac{1-\mu}{2}v_{ij,x\overleftarrow{x}} + \frac{1+\mu}{4}(u_{ij,\overleftarrow{y}x} + u_{ij,y\overleftarrow{x}}) = 0, \tag{2}$$

where $i = \overline{1, n-1}, j = \overline{1, m-1}$ (we use here the generally accepted symbols for finite difference derivatives [3]). The created finite difference scheme (2) approximates the initial differential problem (1) with second order relative to h_x and h_y and represents the system of linear algebraic equations with dimensions $2 \times (n-1) \times (m-1)$. The values $v_{0j}, u_{0j}, v_{nj}, u_{nj}, j = \overline{0,m}$ and $v_{i0}, u_{i0}, v_{im}, u_{im}, i = \overline{0,n}$ are determined from boundary conditions. Taking into account the large dimensions of the system, its solution must be found by means of an iterative method [3].

The developed algorithm is easy to realize and can be applied to the discretization of the regions possessing complicated geometrical structures.

References

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