ROMAI J., **4**, 1(2008), 93–100

# THE IMPACT OF THE STOCHASTIC LIMITER CONTROL ON ECONOMIC DYNAMICS

Adriana-Daniela Gurgui Costea

*Theoretical High School "Ovidius", Constanţa*

adrianagurgui@yahoo.com

**Abstract** The impact of the stochastic limiter on the dynamics of the economy is analyzed. The conventional economic model is modified by adding with or multiplying the characteristic parameters by a random quantity. It is shown that if the limiter contains a stochastic factor, then it is possible for the governing invariant sets of the dynamics generated by the model to change into an equilibrium or a cyclic regime or, even more important, a cyclic regime can turn into a chaotic one. For economists, it is highly important to be able to predict the economic asymptotic behaviors in order to prevent the unwanted situations. That's why MathCAD, which is a very useful and powerful program, is going to be used. The numerical examples and diagrams in the paper are realized by us and they use MathCAD.

**Keywords:** economic dynamics, logistic function, Feigenbaum diagram, limiter control.

**2000 MSC:** 37N40.

## 1. RANDOM REGIME OF THE MODEL

The economic model described by the logistic function was intensively studied in the framework of the topological dynamical systems. However, in reality, the factors determining the dynamics generated by the model are random. In the theory of discrete dynamic systems these random variations are known as *noise*, in analogy to electronic circuits.

In what follows, the limiter factor, the parameter '$h$' is random. We investigate the effects of the stochastic limiter on the dynamics of the economical system. Thus, the model will be modified up to a version that implies the existence of a *noise* in the limiter factor $f_{h,\omega}(x_t) = \min\{ax_t(1-x_t), h(1+\omega\xi_t)\}$, or $f_{h,\omega}(x_t) = \min\{ax_t(1-x_t), h(1+rnd(d))\}$ where $\omega > 0$, is *the intensity* *of noise,* $\xi_t$ is the Gaussian distribution of the variable with the average 0 and the variation 1, and 'rnd' is the random function, $0 < d < 1$. Of course, dynamics are determined by circumstances and can be random. But, as we are mainly interested in the consequences of the *noise* limiter, we shall refer to the last cases.

## 2. RANDOM VARIATIONS

Previously it has been studied the effect of the introduction of a fixed limiter threshold in order to control the dependence of the logistic function. In what follows, we take into consideration the fact that in reality, the values situated under this threshold have a variation which will be stimulated by the introduction of a random element. Two kinds of variations will be analyzed: one, which is stimulated by random numbers characterized by the 'rnd' function and the other which is of a Gaussian type, 'dnorm'. Moreover, there will also be taken into consideration the following cases: the variations will influence $X_t$ function; the variations will influence the parameter $a$. The graphical representations of the two variations are illustrated in fig. 1.

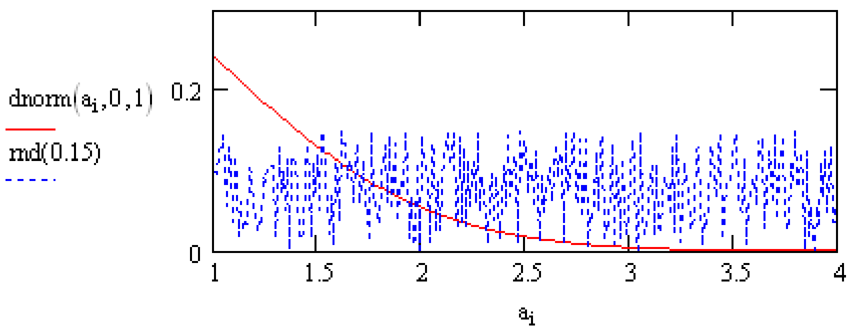



*Fig. 1.* Variations of the limiter threshold.

If **hl** stands for the random form of the threshold, then its expression in terms of the initial determinist threshold **h** will have one of the forms: $h_1 = h(1 + rnd(d))$, or $h_1 = h(1 + \omega dnorm(h, 0, 1))$, where "rnd" is the random function, $0 < d < 1$ is its domain and **dnorm** is the function of normal distribution (of 1 and 0 parameters in the present case).

We are going to illustrate a few representations of the logistic function with both determinist and random threshold in order to establish their influence. In our analysis we also use the numerical series. We start with the logistic function in a chaotic regime corresponding to $a = 3.9$ applying first a deterministic threshold with a superior limiter of $h = 0.9$.

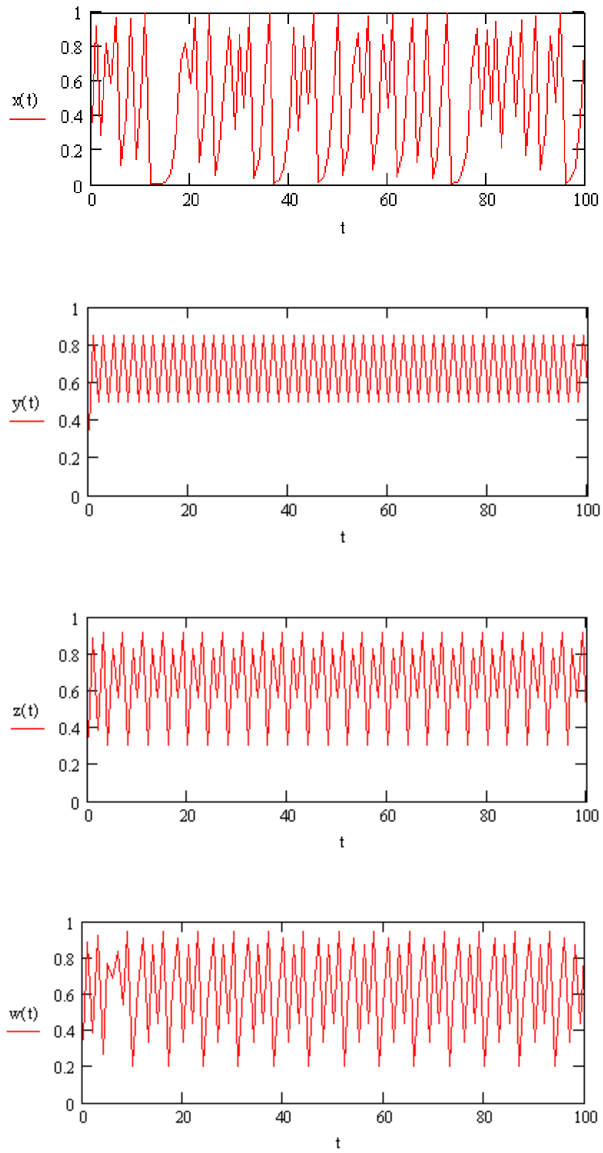



*Fig. 2.* The change of logistic function in: a) chaotic regime; b) with a determinist threshold, h = 0,85; c) with a random threshold h1=h(1+0,4rnd(0,2)); d) with a Gaussian threshold, h2=h(1+0,4dnorm(h,0,1)).

The result is given in fig 2 a). Thus **x(t)** stands for the initial logistic function, **y(t)** for the logistic function limited by a logistic threshold, **z(t)** for the logistic function limited by a random threshold and **w(t)** for the logistic function limited by a Gaussian threshold. Here and in the following by logistic function we mean the motion generated by it.

We notice that, when the determinist threshold is applied, the function changed from its chaotic regime into a cyclic regime of period 2. But, as we can see in the fig 2 c) and in the following table, when a random threshold, whose values vary around the values of the determinist threshold, is applied, the state of the model will change into a cycle of period 4. Also, if a Gaussian threshold is applied, the system changes into a cyclic regime of period 7, (fig 2 d)).

t1 := 90..100

| | x(t1) = 0 | y(t1) = 0 | z(t1) = 0 | w(t1) = 0 |
|---|---|---|---|---|
| 0 | 0.622 | 0.497 | 0.565 | 0.336 |
| 1 | 0.916 | 0.85 | 0.915 | 0.87 |
| 2 | 0.298 | 0.497 | 0.303 | 0.441 |
| 3 | 0.817 | 0.85 | 0.824 | 0.945 |
| 4 | 0.584 | 0.497 | 0.565 | 0.204 |
| 5 | 0.947 | 0.85 | 0.915 | 0.634 |
| 6 | 0.194 | 0.497 | 0.303 | 0.905 |
| 7 | 0.611 | 0.85 | 0.824 | 0.336 |
| 8 | 0.927 | 0.497 | 0.565 | 0.87 |
| 9 | 0.263 | 0.85 | 0.915 | 0.441 |
| 10 | 0.756 | 0.497 | 0.303 | 0.945 |

## 3. FEIGENBAUM DIAGRAMS

In order to study the influence of variations over the limited logistic function (HLC), we shall analyze the influence of variations in the bifurcation Feigenbaum diagram. In this representation we are interested in the bifurcation variable $a_i$, which determines the various regimes of the logistic function (equilibrium, cycles, chaos). For these we are going to define the following variables:

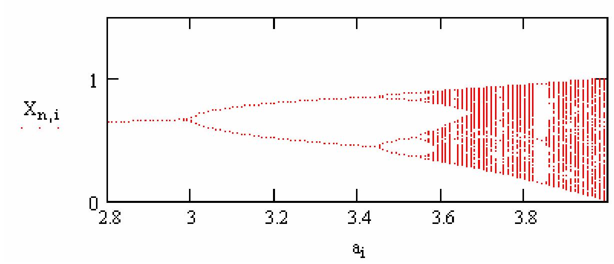
$X_{n,i}$–the basic logistic function ($n$ is the number of iterations and $i$ determines the step of increase of variable $a_i$) (fig 3. a));

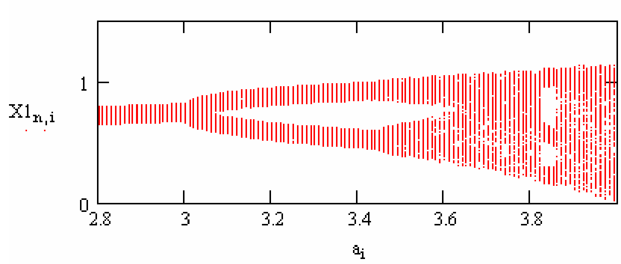
$X1_{n,i}$–the logistic function to which there has been added a random variation (rnd) over $X_{n,i}$ (fig 3. b));

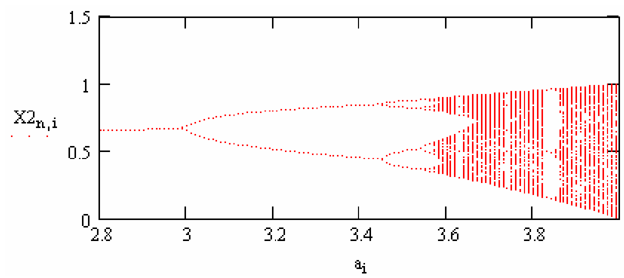
$X2_{n,i}$–the logistic function to which there has been added a Gaussian variation (dnorm) over $X_{n,i}$ (fig 3. c));

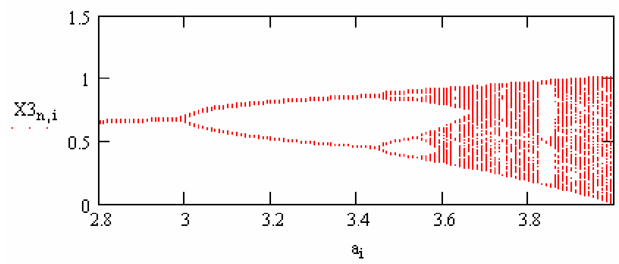
$X3_{n,i}$–the logistic function to which there has been added a random variation (rnd) over $a_i$ (fig 3. d));

$X4_{n,i}$–the logistic function to which there has been added a Gaussian variation (dnorm) over $a_i$ (fig 3. e)).



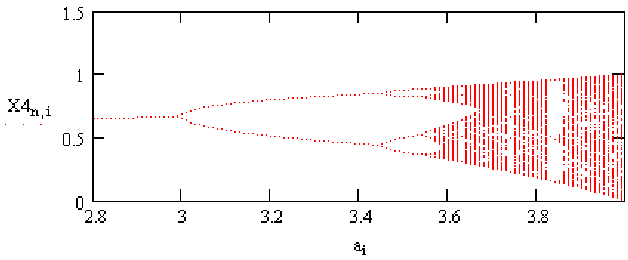



*Fig. 3.* Feigenbaum diagram for the logistic function a) $X_{n,i}$-basic; b) $X1_{n,i}$- to which there has been added a random variation (rnd) over $X_{n,i}$; c) $X2_{n,i}$- to which there has been added a Gaussian variation (dnorm) over $X_{n,i}$; d) $X3_{n,i}$- to which there has been added a random variation (rnd) over $a_i$; e) $X4_{n,i}$- to which there has been added a Gaussian variation (dnorm) over $a_i$

That is: $X_{n+1,i} := a_i X_{n,i}(1 - X_{n,i})$; $X1_{n+1,i} := a_i X_{n,i}(1 - X_{n,i}) + md(0.15)$; $X2_{n+1,i} := a_i X_{n,i}(1 - X_{n,i}) + dnorm(a_i, 0, 1)$; $X3_{n+1,i} := (a_i + md(0.1))X_{n,i}(1 - X_{n,i})$; $X4_{n+1,i} := (a_i + dnorm(a_i, 0, 1))X_{n,i}(1 - X_{n,i})$.

In the case of a random variation, this variation is represented by thick lines. When we take into consideration the Gaussian variation, the dependence of the logistic function is reflected by the change in form and sizes of it. The differences will be illustrated in what follows by calculations. The changes that take place in the case of variations are very important not only for the functions themselves, but also for their averages, calculated for a random number of iterations. We choose a number of 50 iterations between 100 and 150. Define the following averages:

$m\,(n,i)$ the average for 50 iterations of the basic logistic function $X_{n,i}$;

$m1\,(n,i)$ the average for 50 iterations of the basic logistic function $X1_{n,i}$;

$m2\,(n,i)$ the average for 50 iterations of the basic logistic function $X2_{n,i}$;

$m3\,(n,i)$ the average for 50 iterations of the basic logistic function $X3_{n,i}$;

$m4\,(n,i)$ the average for 50 iterations of the basic logistic function $X4_{n,i}$.

where $m\,(n,i) = \frac{1}{50}\sum_{n=100}^{150} X_{n+1,i}$, $\quad m1\,(n,i) = \frac{1}{50}\sum_{n=100}^{150} X1_{n+1,i}$,

$m2\,(n,i) = \frac{1}{50}\sum_{n=100}^{150} X2_{n+1,i}$, $m3\,(n,i) = \frac{1}{50}\sum_{n=100}^{150} X3_{n+1,i}$,

$m4\,(n,i) = \frac{1}{50}\sum_{n=100}^{150} X4_{n+1,i}$. In order to avoid the effect of variations over these averages, we calculate the differences (in absolute value) that follows between these and the basic function average $m\,(n,i)$. The relations of definition are

$$d1(n,i) := |m(n,i) - m1(n,i)|, d2(n,i) := |m(n,i) - m2(n,i)|,$$

$$d3(n,i) := |m(n,i) - m3(n,i)|, d4(n,i) := |m(n,i) - m4(n,i)|.$$

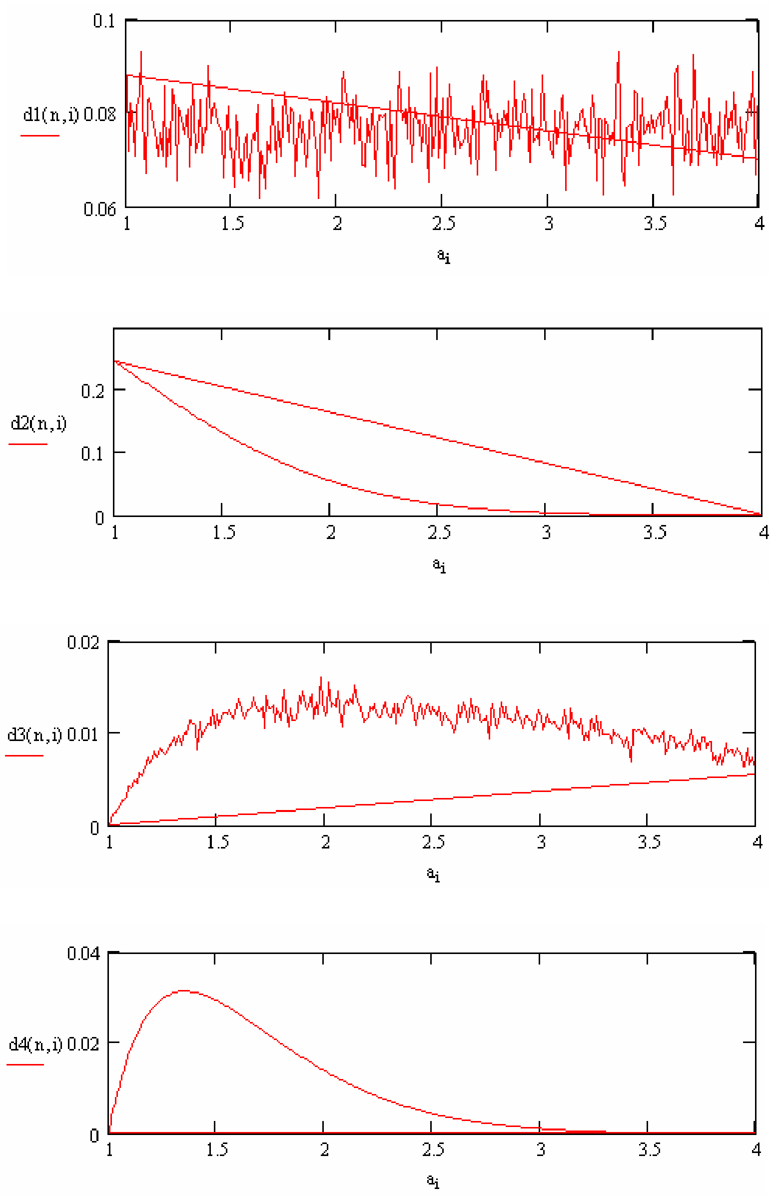



*Fig. 4.* The effects of variations over averages.

## 4. CONCLUSIONS

The mechanisms of hard limiter control are often met in economy. The control generates a stable behavior of the system, even if it is chaotic or periodic. A frequent change in the position of the limiter can appear as a proper strategy which can compensate the new cycles that appear. This strategy will lead to irregular behavior of the system.

Our analysis shows that it is to keep the limiter fixed, changing it only in those moments when the value of the parameter is modified substantially. In this case there will appear adjustable cycles of a smaller period. Out of these the cycle of period one seems to be the best from several points of view in relation to economy. A powerful and permanent control is necessary in order to obtain this regime. Otherwise its behavior will not be at the right standards. When economy is being discussed, these effects could be used as arguments against the control politics. In order to counterbalance such arguments, a simple control politics should be formulated in order to reach the peak of economy.

## References

[1] Cristi, M., *Periodic economic cycles, the effect of evolution towards criticality and crisis*, Journal of Statistical Mechanics, 2005.

[2] Zang, W.-B., *Discrete dynamical systems, bifurcations and chaos in economics*, Amsterdam, 2006.

[3] Hilker, F. M., Westerhoff, F. H. (eds), *Control of chaotic population dynamics: Ecological and economic considerations*, Beiträge des Instituts für Umweltsystemforschung der Universität Osnabrück, **32**, November 2005.

[4] Mathsoft Mathcad User Guide.

[5] Matsoft Engineering & Education.