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ON THE RECONSTRUCTION OF THE PHASE SPACE OF A DYNAMICAL SYSTEM

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Abstract The study of time series with the aim of trying to characterize the underlying dynamic system (or data generating process) is a field of study of great interest in chaos based communications, namely spread spectrum communications with chaotic spreading code. In order to perform any kind of analysis upon a data series first there should be correctly reconstructed the phase space with the key parameters time delay and embedding dimension. In this paper several methods to determine these parameters are presented and their applicability in communications is discussed. Also there is taken into account the influence of noise as the spread spectrum communication have the power spectra lower than the spectra of noise. Also the modulation of informational symbols influence the dynamics of the data, therefore the parameters of interest are much more difficult to determine.

Keywords: time series analysis, chaos based communications.

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1. INTRODUCTION

The concept of chaos is one of the most exciting and rapidly expanding research topics of recent decades. Ordinarily, chaos is disorder or confusion. In the scientific sense, chaos does involve some disarray, but there is much more to it than that. The states of natural or technical systems typically change in time, sometimes in a rather intricate manner. The study of such complex dynamics is an important task in numerous scientific disciplines and their applications as well in communications. Usually the aim is to find mathematical models which can be adapted to the real processes. In the last two decades there were developed nonlinear methods for data analysis that are based on a metric or topological analysis of the phase space of the underlying dynamics or an appropriate reconstruction of it. The subject of discussion in this paper focuses on phase space reconstruction.

Because chaos is very sensitive to initial conditions and chaotic sequences possess a noise-like wide-spectrum characteristic, yet they can be reproduced there very suitable as spreading code in Direct Sequence Spread Spectrum (CD3S) technique. Also, when using chaotic systems, there is no need to assume the randomness, since when observed in a coarse-grained state-space they do not behave randomly, but they do in a long run. Two chaotic trajectories diverge quickly, therefore chaotic systems provide by their nature a large sequence of uncorrelated sequences [13].

Due to several of such special features, including the important diffusion, confusion and mixing (ergodicity) properties [2], chaos-based communication systems can demonstrate some superiority over the conventional DSSS codes.

The proposed structure of a CD3S transmitter [3] :

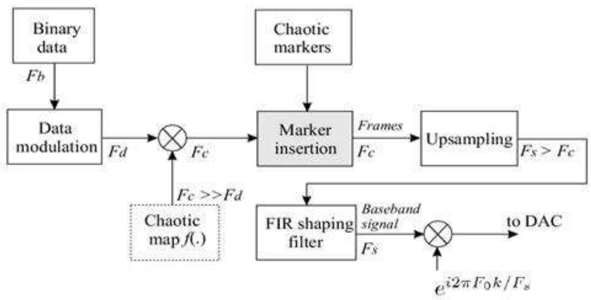

Fig. 1. Structure of a DSSS transmitter

Direct-sequence spectra vary somewhat in spectral shape, depending on the actual carrier and data modulation used. In fig. 1 is employed a binary phase shift keyed (BPSK) signal, which is the most common modulation type used in direct sequence systems. Afterward the spectra is spread along a much larger bandwidth and upsampled in order to obtain a sampled signal with a "continuous" dynamics.

In this paper the possibility to reconstruct the phase space of CD3S received signal is investigated. In the next section the methods employed with examples for a classical dynamical system, Lorenz, are presented and, afterward, the methods are applied to a modulated signal.

2. METHOD OF DELAYS

Systems of very different kinds, from very large to very small time-space scales can be modeled mathematically by (deterministic) differential equations.

Let the possible states of a system be represented by points in a finite-dimensional phase space, $\Re^m$. The transition form a system state from t_1 at time t_2 is a deterministic rule $T_{t_2-t_1}$[11]. For **continuous-time dynamics** with variables $\mathbf{x}(t) = [x_1(t), x_2(t), ..., x_m(t)]$, there can be defined a set of m ordinary differential equations

$$\dot{\mathbf{x}}(t) = \frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{G}(x) \tag{1}$$

where the vector field $\mathbf{G}(x)$ is always taken to be continuous in its variables and also taken to be differentiable as often as needed. The family of transition rules T_t, or its realization in the forms (1), are referred to as a *dynamical system.* Formally, a dynamical system is given by $\mathbf{x}(t)$ in a m dimensional space, which is called the *phase space*, for continuous time and a time evolution law (1).

Usually the observation of a real world process does not allow all possible state variables. Either not all state variables are known or not all of them can be measured. Most often only one observation is available

$$u(t) = h(x(t)) \tag{2}$$

where $h(\cdot)$ is the *measurement function.* Since the measurement results in discrete time series, the observations will be referred to as $u(k) = x(t_0 + kT)$, T being the sampling rate of the measurement.

As stated before, trying to reconstruct the state space of a system from information on scalar measurements $u(k) = x(t_0 + kT)$ is equivalent to finding a connection between the derivatives and the states variables, namely the differential equations which produced the observations. This is named the method of *derivatives coordinates.* Since this methods employs high order coordinates, the presence of noise can affect the reconstruction and therefore this method is not very useful for experimental data, so the mostly approached method is the *method of delays* (MOD) which is the point of the further discussion.

This method was first introduced into dynamical systems theory independently by Packard *et al* [8],and by David Ruelle and Takens [12] and it consists in the fact there really is no need of derivatives to form the system coordinates in which to capture the structure of orbits in phase space, but that there can be used directly the lagged samples.

The more rigorous formulation of this theorem is given as: consider K a smooth $\left(C^2\right)$ m-dimensional manifold that constitutes the original state space of the dynamical system under investigation and let $\phi^t : K \to K$ be the corresponding flow. Suppose that one can measure some scalar quantity $u\left(t\right) = h\left(x\left(t\right)\right)$ that a given by the measurement function $h : K \to \Re$, where $u\left(t\right) = \phi^t\left(u\left(0\right)\right).$ Then one may reconstruct a *delay coordinates map*:

$$F : K \to \Re^m$$

$$u \to \mathbf{y}(k) = [u(k),\, u(k+T),\, u(k+2T), ..., u(k+(m-1)J] \tag{3}$$

$$k = \overline{1, M} M = N - (m-1)J$$

that maps a state u from the original state space K to a point $\mathbf{y}$ in the *reconstructed state space* $\Re^m$, where m is the embedding dimension, J gives the time lag to be used [9], and N gives the number of measurements.

The MOD reconstructs the attractor dynamics by using delay coordinates to form multiple state-space vectors, $\mathbf{y}\left(\mathbf{i}\right)$. That is the reconstructed trajectory, $\mathbf{Y}$, is given by

$$\mathbf{Y} = \left[\mathbf{y}\left(\mathbf{1}\right)\,\mathbf{y}\left(\mathbf{2}\right)\ldots\mathbf{y}\left(\mathbf{M}\right)\right]^{\mathbf{T}}. \tag{4}$$

If the time series represents a continuous flow with samples taken every Δt seconds, then the delay time τ is the time period between successive of each of the embedding space vector. τ is considered an integer multiple of the sampling period and it can be expressed as $\tau = J \cdot T$. Here there should be stressed the difference between time delay τ and time lag J.

This is possible because in nonlinear systems the multiple dynamical variables interact with one another [1]. This was demonstrated numerically by Packard *et al* [8] and was proved by Takens [12]. The method of delays is the most widespread approach because it is the most straightforward and the noise level is constant for each delay component. Still this methods has a major drawback: the quality of reconstruction depends upon choosing the appropriate embedding parameters, the dimension m and the time delay J.

2.1. ESTIMATING TIME DELAY

As suggested, when employing the MOD, the only parameter that affects the quality of reconstruction is the embedding dimension. Also there should be stressed that it assumes the availability of an infinite amount of noiseless data. Instead, in real experiments, finite noisy data sets are used, therefore proper care should be taken when choosing the time delay (lag).

In estimating the value of τ many methods were considered but none is universal and when selecting it, two major problems should be considered: *redundancy* and *irrelevance*, as called by Casdagli *et al.* in *[5]*. In order to better exemplify the presented phenomena the Lorenz attractor in fig. 2 is considered.

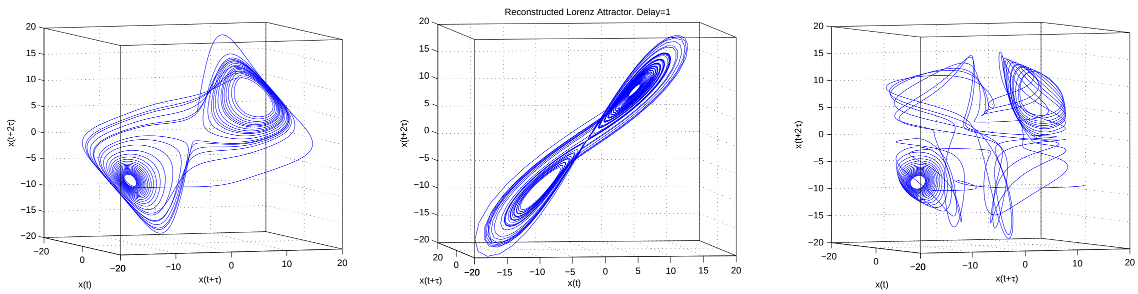

Fig. 2. Reconstructed Lorenz attractor with different time delays. Noiseless data set. $\sigma = 10$, $R = 28$, $b = \frac{8}{3}$, sampling period $T = 0.01$, number of samples $N = 3500$, $(a)\, \tau = 17T$, $(b)\, \tau = T$, $(c)\, \tau = 51T$.

The first one is related to the choice of τ as small as possible. In this case the consecutive measurements of the reconstructed vectors will give nearly the same results. Hence, the topological vectors constructed via the method of delays, will be stretched along the main diagonal, or line of identity, in the m-dimensional embedding space, leading to a resemblance for the reconstructed attractor with dimension close to one and thus the analysis of the picture of the attractor will be very difficult.

The second problem, irrelevance, appears with the choice of τ too large. In this case the reconstructed vectors become totally uncorrelated and the extraction of any information from this phase space picture becomes impossible, therefore resulting a random distribution of points in the embedding space.

Considering all mentioned above, τ should make independent each component in the reconstructed vector.

Ideally a method to estimate the delay should be computationally efficient, work well with noisy data and lead to consistent, accurate estimate of key descriptors of the original attractor.

Average displacement method. There are several criteria for the selection of the time delay τ, the main of them being the autocorrelation method. The main disadvantage of this methods is that it gives reasonable time delay only for two dimensional systems and in the literature are presented several criteria that were proved to be very sensitive to the employed dynamical system. Another popular method is the first local minimum of the delayed mutual information. Like the autocorrelation method, this approach works well for low dimensional dynamical systems and, in addition, it is sensitive to the number of bins employed for the segmentation of coordinates. Therefore there should be considered a method that offers different results for different number of dimensions. The investigated method, named average displacement, was introduced by Rosenstein and his colleagues [10] and quantifies the expansion of the attractor from the main diagonal. It is intended to overcome the drawbacks of autocorrelation.

For various embedding dimensions the *Average Displacement Method* (AD) can be computed as a function of τ such that

$$\langle S_m(J)\rangle = \frac{1}{M}\sum_{i=1}^{M} \|Y_i^{\tau} - Y_i\|, \tag{5}$$

where the upper scripts denote the delay between successive embedding components[10]. If they are used, the scalar time series (5) can be rewritten as

$$\langle S_m(J)\rangle = \frac{1}{M}\sum_{i=1}^{M} \sqrt{\sum_{j=1}^{m-1} [x_{i+jJ} - x_i]^2}. \tag{6}$$

The average displacement is useful for quantifying the decrease in redundancy error with increasing τ. As the time delay increases from zero the average displacement increases accordingly until it reaches a plateau. When increasing the value of m, the time until $\langle S_m\rangle$ reaches a plateau is shorter, therefore a constant embedding window can be maintained.

The two dimensional phase portrait for the Lorenz time series with the time delay generated by the average displacement method can be seen in fig.3.

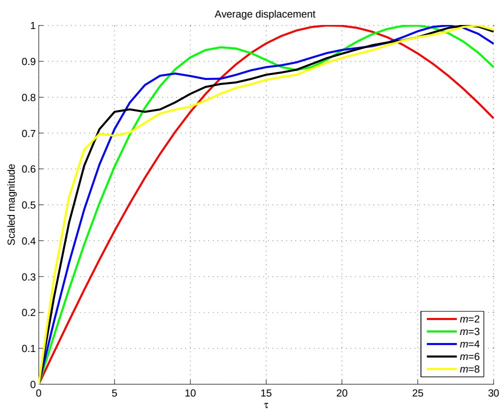

Fig. 3. Average displacement for Lorenz dynamical system and 2D attractor reconstruction with the provided time delay. $\sigma = 10$, $R = 28$, $b = \frac{8}{3}$. Initial conditions $x_0 = \frac{1}{10}$, $y_0 = -\frac{1}{5}$, $z_0 = \frac{3}{10}$, sampling period $T = 0.02$, number of samples $N = 3500$. (b) $m = 3$, $\tau = 9T$.

Based on empirical results, the authors suggested choosing time delay, "*as the point where the slope first decreased to less than 40% of its initial value*" [10]. Still when the wave shapes reach a certain saturation there are some waviness, and using the changing slope to determine the time delay may introduce some errors.

There can be seen an anomalous jump from $J = 0$ to $J = 1$, due to the fact that when using a zero time delay the phase-space vector is aligned with the line of identity.

Multiple correlation method. Multiple autocorrelation approach is derived from autocorrelation and *Average Displacement Method*. Considering (5), the square average displacement for a chaotic time series, $\{x_i\}$ in an m-dimensional space can be written as

$$\left\langle S_m^2\left(J\right)\right\rangle = \frac{1}{M}\sum_{i=1}^{M}\|y_i^\tau - y_i\|^2, \tag{7}$$

such that (6) becomes

$$\left\langle S_m^2\left(J\right)\right\rangle = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{m-1}\left[x_{i+jJ} - x_i\right]^2. \tag{8}$$

Before deriving a relationship between $\left\langle S_m^2\left(J\right)\right\rangle$and the autocorrelation, we note that the expression for a finite data set for autocorrelation is given by

$$R_{xx}\left(J\right) \approx \frac{1}{N-J}\sum_{i=1}^{N-J} x_i \cdot x_{i+J}. \tag{9}$$

Extending the left-hand side of (8) and ignoring the errors caused by the border data we obtain

$$\left\langle S_m^2\left(J\right)\right\rangle = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{m-1}\left[x_{i+jJ}^2 - 2x_i\cdot x_{i+jJ} + x_i^2\right] = 2\left(m-1\right)E - 2\sum_{j=1}^{m-1}R_{xx}\left(jJ\right), \tag{10}$$

where $E = \frac{1}{M}\sum_{i=1}^{M} x_i^2 = \frac{1}{M}\sum_{i=1}^{M} x_{i+jJ}^2$, for $1 \leq j \leq m-1$ and $R_{xx}^m\left(J\right) = \sum_{j=1}^{m-1} R_{xx}\left(jJ\right)$.

As it was considered by the authors of [7], the multiple autocorrelation delay can be described as: select the corresponding time as the time delay τ when the value of $R_{xx}^m\left(J\right)$ decreased to the $1/e^{-1}$ times of its initial value.

Moreover, when replacing the autocorrelation method with the non bias multiple autocorrelation we have

$$C_{xx}^{m}(J) = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{m-1}(x_i - \overline{x})(x_{i+jJ} - \overline{x}) = R_{xx}^{m}(J) - (m-1)(\overline{x})^2 . \quad (11)$$

In fig. 4 the results for x variable of Lorenz system perturbed with noise are illustrated.

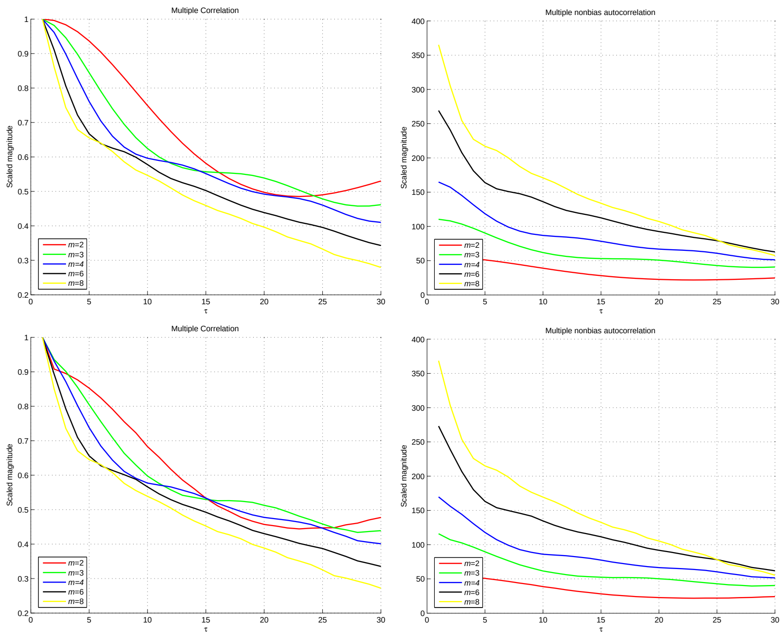

Fig. 4. Multiple autocorrelation for Lorenz dynamical system. $\sigma = 10$, $R = 28$, $b = \frac{8}{3}$. Initial conditions $x_0 = \frac{1}{10}$, $y_0 = -\frac{1}{5}$, $z_0 = \frac{3}{10}$, sampling period $T = 0.02$, number of samples $N = 3500$.

This algorithm can be regarded as an extension of the autocorrelation approach in the high order dimension.

2.2. ESTIMATING EMBEDDING DIMENSION

Mańé and Takens theorem guarantees that the reconstructed dynamics, if properly embedded, is equivalent to the dynamics of the true, underlying system in the sense that dynamical invariants, such as generalized dimensions and the Lyapunov spectrum, are identical.

There were different methods proposed for determining the minimum embedding dimension, but the most commonly used is the geometrical approach, false nearest neighbors (FNN). The idea of this method is quite intuitive. Suppose that the minimal embedding dimension for a given time series $\{x_i\}$ is m_0. Suppose that one embeds the time series in an m-dimensional space with $m < m_0$. Due to this projection the topological structure is no longer preserved. Points are projected into neighborhoods of other points to which they would not belong in higher dimensions. These points are called false neighbors. If now the dynamics is applied, these false neighbors are not typically mapped into the image of the neighborhood, but somewhere else, so that the average "diameter" becomes quite large.

The idea of the algorithm false nearest neighbors is the following: since each point y_i from the reconstructed vector as in (2) has a nearest neighbor y_j with the nearness in the sense of some distance function (in the simulations the Euclidean distance is used), then

$$\begin{aligned} R_m(i)^2 = [&\left(x_i - x_{n(i,m)}\right)^2 + \left(x_{i+J} - x_{n(i,m)+J}\right)^2 + \dots \\ &+ \left(x_{i+(m-1)J} - x_{n(i,m)+(m-1)J}\right)^2], \end{aligned} \tag{12}$$

where $R_m(i)$ is rather small when there is a large amount of data and can be approximated by $1/N^{\frac{1}{m}}$, where N represents the number of samples.

In the dimension $m+1$ the nearest neighbor distance is changed due to the $(m+1)^{st}$ coordinates x_{i+mJ} and $x_{n(i,m)+mJ}$ and R_{m+1} can be computed as

$$R^2_{m+1}(i) = R^2_m(i) + \left[x_{i+mJ} - x_{n(i,m)+mJ}\right]^2. \tag{13}$$

If $R_{m+1}(i,j)$ is too large, there can be presumed that this happens because the near neighborliness of the two points being compared is due to the projection from some higher dimensional attractor down to dimension m. Some threshold is required to decide when neighbors are false. Then, if

$$\frac{\left|x_{i+mJ} - x_{n(i,m)+mJ}\right|}{R_m(i)} > R_T, \tag{14}$$

the nearest neighbor at time point j are declared false. In practice for values of R_T in the range $10 \leq R_T \leq 50$ the number of false neighbors identified by this criterion is constant.

For the Lorenz attractor, the embedding dimension, where the percentage of false neighbors drops to zeros, is 3, while the sufficient condition from the embedding Takens theorem is 5.

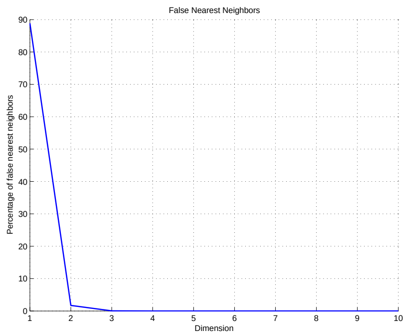

Fig. 5. Percentage of false nearest neighbors as a function of embedding dimension for clean Lorenz data. $\sigma = 10$, $R = 28$, $b = \frac{8}{3}$. Initial conditions $x_0 = \frac{1}{10}$, $y_0 = -\frac{1}{5}$, $z_0 = \frac{3}{10}$, sampling period $T = 0.02$, time lag $J = 9T$, number of samples $N = 3500$.

The so called flaw of this method is that it involves the time lag, and without a proper time delay J, the embedding dimension can not be estimated accurately.

2.3. EMBEDDING WINDOW

The embedding window is defined as the length of the interval spanned by the first and last delay coordinate

$$\tau_w = \tau \cdot (m - 1)\,. \tag{15}$$

The parameter is thoroughly discussed in [6] and it is shown to be more useful, since it determines the amount of information passed form the time series to the embedding vectors, than the dependent parameters: m the embedding dimension and τ time delay.

There are several methods employed in order to establish the time window and the most of them are related to the saturation of the correlation dimension.

As shown in [4, 6] the determination of minimal necessary embedding dimension is strongly dependent upon the choice of the time delay. As one expects the embedding dimension decreases when increasing the time delay, as the delayed coordinates are more and more independent. As a consequence, a wise solution would be to apply the false nearest neighbor method, but considering proper time delays for each dimension. Therefore, the time delay should be established for each $d = 2, 3, ...$ with one of the prior learned approaches: average displacement, multiple autocorrelation or multiple non-bias autocorrelation.

3. EXPERIMENTAL RESULTS

A particularly interesting candidate for **discrete time** chaotic sequences generators is the family of Chebyshev polynomials, whose chaoticity can be verified easily and many other properties of which are accessible to a rigorous mathematical analysis. The independent binary sequences generated by a chaotic Chebyshev map were shown to be not significantly different from random binary sequences. For this reason, a k^{th}-order Chebyshev map is employed in the following discussion for masking the information. This map is defined by

$$\begin{gathered} x_{k+1} = F\left(x_k\right) = T_p\left(x_k\right), \\ T_0 = 1, \\ T_1 = x, \\ T_{p+1}\left(x\right) = 2xT_p\left(x\right) - T_p\left(x\right), \end{gathered} \tag{16}$$

p being the order of the polynomial.

Consider a data signal

$$D = [1, -1, 1, 1, -1, 1, 1, 1, -1, 1, 1, -1, -1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1]\,. \tag{17}$$

After the spreading process, a raised cosine shaping filter with the roll off factor $\alpha = 0.3$, that introduces a delay of 3 samples, is applied. Considering the sampling frequency $F_e = 5F_c$, after this shaping the signal will have a more smooth dynamics suitable to methods that require a continuous time systems (smooth dynamics). This is equivalent to interpolating with a rate $R = 5$. These algorithms can be applied to the data as they offer a quite good estimation for the time lag when discussing continuous data. However, when considering discrete chaotic time series, they cannot process it accurately, because the sampling spacing of the discrete time series is too large, therefore the maps behave like random series.

The delay introduced by the filter was discarded.

Trying to reconstruct the phase space of the signal the following results for the time delay were obtained

Table 1: Estimated time lag for the filtered spread spectrum sequence with spreading sequence 2^{nd} order Chebyshev polynomial

	Processing gain $G = 31$			Processing gain $G = 63$			Processing gain $G = 1024$		
Method	$m = 2$	$m = 3$	$m \geq 4$	$m = 2$	$m = 3$	$m \geq 4$	$m = 2$	$m = 3$	$m \geq 4$
Average Displacement	6	4	3	6	4	3	6	4	3
Multiple Autocorrelation	4	3	2	4	3	2	4	3	2
Multiple non bias Autocorrelation	4	3	2	4	3	2	4	3	2

For the estimation of the embedding dimension we did not consider the whole data set as both implemented methods require a great computational cost; instead an amount of $N = 1500$ was considered, and the segment of data was chosen randomly. Taking into account the time lags estimated before and the previous remarks, the minimum embedding dimension was determined to be $m = 3$.

From the previous work we saw that trying to reconstruct the phase space in the presence of noise can be troubling especially in the case of a low signal to noise ratio which is the case in the most situations that deal with CD3S. Modulation, as proved before, makes even more difficult. Anyway, knowing that the expected signal should be low dimensional, one could guide oneself when choosing the appropriate m. Also there should be kept in mind that a too high embedding dimension can cause spurious correlation. Again for a low Signal to Noise Ratio (SNR) the dimension will be inflated. Therefore a compromise should be made.

Table 2: Estimated time lag for noisy CD3S with spreading sequence 2nd order Chebyshev polynomial

Processing gain $G = 31$	SNR=30 dB			SNR=20 dB			SNR=10 dB			SNR=0 dB			
Method	m=2	m=3	m≥4	m=2	m=3	m≥4	m=2	m=3	m≥4	m=2	m=3	m≥4	m≥5
Average Displacement	6	4	3	6	4	3	6	4	3	7	5	4	3
Multiple Autocorrelation	4	3	2	4	3	2	4	3	2	2			
Multiple non bias Autocorrelation	4	3	2	4	3	2	4	3	2	2			

30	Time lag J	FNN
20	5	4
10	$[6,4,3,\ldots]$	5
0	$[4,3,2,\ldots]$	3
20	5	5
	$[6,4,3,\ldots]$	4
	$[4,3,2,\ldots]$	3
10	5	4
	$[6,4,3,\ldots]$	4
	$[4,3,2,\ldots]$	3
0	2	4
	3	4
	$[7,5,4,\ldots]$	4

The table above presents the minimum embedding dimension estimated by False Nearest Neighbor method for noisy CD3S spread with spreading sequence 2^{nd} order Chebyshev polynomial

From the previous tables one can draw the conclusion that the *Average Displacement Method* yields the most reliable results when estimating the time delay, even in noisy environment for all investigated systems. We allege that as until a $SNR = 0\,dB$ these two methods are quite robust to noise (white Gaussian noise). Still there cannot be designated one method superior to another, because in order to draw such a conclusion one should investigate more replications of systems of interest with different data length.

As far as the embedding dimension is concerned, the results coincide with expectations, meaning that as the signal to noise ratio decreases the embedding dimension increases. Also both methods show dependence upon the choice of time lag, and only in case of the an appropriate J the embedding dimension will be as expected.

4. CONCLUSIONS

In this paper several methods for reconstructing the phase space of a dynamical system from the measurements of one variable were presented. Even though these methods were largely used in other search domains as finances (critical regime in financial indices) and medical engineering (cardiology, neurology etc.), these methods find their applicability in spread spectrum communications, were the modulation alters the natural dynamics of data. The presence of noise was considered too. There can be seen that the results conveyed are different from noiseless data: the time delay decreases as the data becomes more and more uncorrelated and the embedding dimension increases as the noise tends to fill itself the phase space.

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