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# ON $CH$-QUASIGROUPS OF FINITE SPECIAL RANK

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**Abstract** $CH$-quasigroups of finite special rank are characterized by means of various non-medial (or medial) subquasigroups and by means of various non-Abelian (or Abelian) subgroups of their multiplication group.

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## 1. INTRODUCTION

By analogy with the group theory [1], the special rank of a quasigroup $Q$ is called the least positive number $rQ$ with the following property: any finitely generated subquasigroup of the quasigroup $Q$ can be generated by $rQ$ elements; if there are not such numbers, then we suppose that $rQ = \infty$.

The theory of commutative Moufang loops (CML's) is one of the deepest and subtlest areas of the present-day theory of quasigroups and loops. It is certainly one of the most interesting areas, mainly because of its many connections with other topics, in particular, with $CH$-quasigroups [2]-[4]. Such quasigroups are investigated in this paper by means of CML. Concretely, the various equivalent finiteness conditions of special rank of CML and their multiplication groups from [5] are transferred on $CH$-quasigroups.

Let $(Q, \cdot)$ be a non-empty set $Q$ provided with a binary operation $(x, y) \rightarrow x \cdot y$. One says that $(Q, \cdot)$ is a *TS-quasigroup* if any equality of the form $x \cdot y = z$ remains true under all permutations of $x, y, z$ (equivalently $xy = yx$ and $x{\cdot}xy = y$). A quasigroup is said to be *medial* if $xu{\cdot}xy = xv{\cdot}uy$ holds. $(Q, \cdot)$ is a *$CH$-quasigroup* iff $(Q, \cdot)$ is a $TS$-quasigroup such that $xy \cdot xz = xx \cdot yz$ holds. The last condition is equivalent with the following condition: every subquasigroup generated by three elements is medial. For a loop, the identity $xy \cdot xz = xx \cdot yz$ characterizes the CML [2]-[4].

Given four elements $x_1, x_2, x_3, k$ of any quasigroup $(Q, \cdot)$ *the mediator of* $x_1, x_2, x_3$ *with respect to* $k$ is the element $a$ of $Q$ uniquely determined by the equality $x_1x_2 \cdot kx_3 = x_1a \cdot x_2x_3$. We denote $a = [x_1, x_2, x_3]_k$. We say that *the mediators of weight* $i$ *with respect to* $k$ are the mediators of the form $[x_1, x_2, \dots, x_{2i+1}]_k$ defined inductively by $\alpha_1 = [x_1, x_2, x_3]_k$ and $\alpha_{i+1} = [\alpha_i, x_{2i+2}, x_{2i+3}]_k$. A quasigroup $Q$ is called medially nilpotent of class $n$ if it satisfies the identity $[x_1, x_2, \dots, x_{2n+1}]_y = y$, but does not satisfy the identity $[x_1, x_2, \dots, x_{2n-1}]_y = y$.

The *multiplication group* $\mathfrak{D}$ of a $CH$-quasigroup $(Q, \cdot)$ is the group generated by all *translations* $L(x)$, where $L(x)y = xy$, $x \in Q$.

**Theorem.** *For an arbitrary non-medial* $CH$*-quasigroup* $Q$ *with the multiplication group* $\mathfrak{D}$ *and its subgroup* $\mathfrak{D}^0$*, consisting of products of even number of translations* $L(x)$*,* $x \in Q$*, the following statements are equivalent:*

1) $Q$ *has a finite special rank;*

2) *if* $Q$ *contains a medially nilpotent subquasigroup of class* $n$*, then all its subquasigroups of this type have a finite special rank;*

3) *at least one maximal medial subquasigroup of* $Q$ *has a finite special rank;*

4) *non-normal medial subquasigroups of* $Q$ *have a finite special rank;*

5) *normal subquasigroups of* $Q$ *have a finite special rank;*

6) $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) has a finite special rank;*

7) *all normal subgroups of* $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) have a finite special rank;*

8) *all non-normal Abelian subgroups of* $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) have a finite special rank;*

9) *at least one maximal Abelian subgroup of* $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) has a finite special rank;*

10) *if* $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) contains a nilpotent subgroup of class* $n$*, then all its subgroups of this type of* $\mathfrak{D}$ *have a finite special rank;*

11) *if* $\mathfrak{D}$ *(respect.* $\mathfrak{D}^0$*) contains a solvable subgroup of class* $s$*, then all its subgroups of this type of* $\mathfrak{D}$ *have a finite special rank.*

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