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NEW FEATURES IN SIMULATING THE BEHAVIOR OF TURBULENT FLOWS

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Abstract This paper continues the previous work in the turbulent mixing field. The mixing theory is a modern theory in the field of flow kinematics. Its mathematical methods and techniques developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called far from equilibrium systems. These are widespread between the models of excitable media.

Studying a mixing for a flow implies the analysis of successive stretching and folding phenomena for its particles, the influence of parameters and initial conditions. In the previous works, [1,2], the 3D non-periodic models exhibited a quite complicated behavior. In agreement with experiments [4], they involved some significant events - the so-called rare events. The variation of parameters had a great influence on the length and surface deformations. The 2D (periodic) case was simpler, but significant events can issue for irrational values of the length and surface unit vectors, as was the situation in 3D case. The periodic case analysis was recently started, in order to establish some statistic features of the mixing flow behavior. The aim of this paper is to get a new approach of numeric simulation of turbulent mixing. Namely, specific plotting new procedures of Maple11 soft are tested, such as the Interactive Plot Builder. A graphical comparative analysis between the discrete and the continuous time case is realized, for the same unit vector irrational values. The results should be used for further numerical study.

Keywords: turbulent mixing, numeric simulation.

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1. INTRODUCTION

In turbulence theory, two important features are generally distinguished: the transition theory from smooth laminar flows to chaotic flows (characteristic to turbulence) and the statistic studies of the completely developed turbulent systems.

The statistical point of view of a flow has the following representation

$$x = \Phi_t(X) \quad \text{with} \quad X = \Phi_t(t=0)(X) \tag{1}$$

which must be of class C^k. From the dynamic point of view the map

$$\Phi_t(X) \longrightarrow x \tag{2}$$

is a diffeomorphism of class C^k and (1) must satisfy the relation

$$0 < J < \infty,\ J = \det\left(\frac{\partial x_i}{\partial X_j}\right),\ J = det(D\Phi_t(X)) \tag{3}$$

where D denotes the differentiation with respect to the reference configuration, in this case X. The relation (3) implies two particles, X_1 and X_2, which occupy the same position x at a moment. Non-topological behavior (like break up, for example) *is not allowed.*

Define the basic measure for the deformation, namely the *deformation gradient*, **F**

$$\mathbf{F} = \left(\nabla_X \Phi_t(\mathbf{X})\right)^T,\ F_{ij} = \left(\frac{\partial x_i}{\partial X_j}\right),\ \text{or}\ \mathbf{F} = D\Phi_t(\mathbf{X}) \tag{4}$$

where ∇_X denotes differentiation with respect to X. According to (3), **F** is not singular. The basic measure for the deformation with respect to x is the *velocity gradient* (∇_x denote differentiation with respect to x).

By differentiation of x with respect to X there are obtained the basic deformation for a material filament, and for the area of an infinitesimal material surface.

In what follows we focus on the basic deformation measures: the *length deformation* λ and *surface deformation* η, defined by the relations [5]

$$\lambda = (C : MM)^{\frac{1}{2}}\,,\ \eta = (\det F)\cdot\left(C^{-1} : NN\right)^{\frac{1}{2}}\,, \tag{5}$$

with $\mathbf{C}(= \mathbf{F}^T\mathbf{F})$ the Cauchy-Green deformation tensor, and the length and surface vectors M, N defined by

$$\mathbf{M} = d\mathbf{X}/\left|d\mathbf{X}\right|\,,\ \mathbf{N} = d\mathbf{A}/\left|d\mathbf{A}\right|. \tag{6}$$

The relation (5) has the following scalar form

$$\lambda = C_{ij}\cdot M_i\cdot N_j,\quad \eta = (\det F)\cdot\left(C_{ij}^{-1}\cdot N_i\cdot N_j\right) \tag{7}$$

with $\sum M_i^2 = 1,\ \sum N_j^2 = 1.$

The deformation tensor $\mathbf{F}$ and the associated tensors $\mathbf{C}$, $\mathbf{C}^{-1}$ represent the basic quantities in the deformation analysis for the infinitesimal elements.

In this framework, the mixing concept implies the stretching and folding of the material elements. If at an initial location P there is a material filament dX and an area element dA, the specific length and surface deformations are given by the relations

$$\frac{D\left(\ln\lambda\right)}{Dt} = \mathbf{D}:\mathbf{mm},\quad \frac{D\left(\ln\eta\right)}{Dt} = \nabla\mathbf{v} - \mathbf{D}:\mathbf{nn}, \tag{8}$$

where $\mathbf{D}$ is the deformation tensor, obtained by decomposing the velocity gradient in its symmetric and non-symmetric part.

2. THE PERIODIC 2D MIXING MODEL. COMPARATIVE ANALYSIS

Studying a mixing for a flow implies the analysis of successive *stretching* and *folding* phenomena for its particles, the influence of parameters and initial conditions [4,5]. In the previous works, the study of the 3D non-periodic models exhibited a quite complicated behavior [1]. Recently the behavior of the mixing flow in 2D case was started, both for periodic [3] and non-periodic case [2]. In what follows we continue this analysis, with a different point of view. Some irrational values of the length and surface unit vectors are also chosen, like in previous works, in order to search some significant events, and compare them to 3D case.

Let us start with the following periodic 2D mixing model [5]

$$v_x = -\varepsilon \cdot x, \tag{9}$$

$$v_y = \varepsilon \cdot y, \quad 0 < t < T_{ext}$$

$$v_r = 0, \tag{10}$$

$$v_\theta = -\omega\left(r\right), T_{ext} < t < T_{ext} + T_{rot}$$

with $-1 < K < 1$, $0 < G < 1$.

In the above relations, the first part represents the extensional part and the second, the rotational part of the so- called t*endril-whorl (TW) model*.

As shown in [5], two-dimensional flows increase their length by forming two basic kinds of structures: *tendrils* and *whorls* and their combinations. The tendril-whorl flow (TW) introduced by Khakhar, Rising and Ottino (1987) is a discontinuous succession of extensional flows and twist maps. Even the simplest case is complex enough. The physical motivation for this flow is that locally, a velocity field can be decomposed into extension and rotation.

The above model is the simplest case of the TW model, where the velocity field over a single period is given by its extensional and rotational part, where T_{ext} denotes the duration of the extensional component and T_{rot} the duration of rotational component.

For the moment we study only the extensional component. So, for the system (9) the solution of the model

$$x = X \cdot \exp\left(-\varepsilon \cdot T_{ext}\right), \tag{11}$$

$$y = Y \cdot \exp\left(\varepsilon \cdot T_{ext}\right)$$

leads to the gradient deformation F and the Cauchy-Green tensors C, C^{-1} of quite simple forms. Therefore the deformations in length and surface λ^2 and η^2 has the following similar forms [3]

$$e_\lambda = 2\varepsilon \cdot \left(1 - \frac{2\exp\left(-2\varepsilon T_{ext}\right) \cdot M_1^2}{\exp\left(-2\varepsilon T_{ext}\right) \cdot M_1^2 + \exp\left(2\varepsilon T_{ext}\right) \cdot M_2^2}\right), \tag{12}$$

$$e_\eta = 2\varepsilon \cdot \left(1 - \frac{2\exp\left(-2\varepsilon T_{ext}\right) \cdot N_2^2}{\exp\left(-2\varepsilon T_{ext}\right) \cdot N_2^2 + \exp\left(2\varepsilon T_{ext}\right) \cdot N_1^2}\right), \tag{13}$$

where the condition $M_1^2 + M_2^2 = 1$, $N_1^2 + N_2^2 = 1$ must be satisfied for the length and surface unit vectors.

In what follows, the behavior of e_λ and e_η is analysed, using a new Maple11 tool, namely *Interactive Plot Builder.* Its calling sequence is the following

interactive(expr, variables), with the parameters:
expr - (optional) expression;
variables - (optional) expression of the form variables=varset.

The *interactive* command is part of the plots package. It allows us to build plots interactively.

If *expr* is an algebraic expression or a list, it is taken to be a single expression to be plotted. If expr is a set of algebraic expressions or lists, these are taken to be separate expressions to be plotted in the same graph.

If present, the *variables* option provides a complete list of the variables used in plotting.

The *Interactive Plot Builder* generates interactive plots, using *continuous time intervals.* Therefore the analysis of the mixing behavior produces useful informations, comparing to discrete case [2,3].

The recent experience revealed interesting events for irrational values of the length and surface unit vectors. Therefore, some irrational unit vectors values were chosen for this analysis, namely

(a) $(M_1, M_2) = (N_1, N_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; (b) $(M_1, M_2) = (N_1, N_2) = \left(\frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$; (c) $(M_1, M_2) = (N_1, N_2) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

For the parameter $0 < \varepsilon < 1$ there were considered two values: $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.08$. Thus, the following situations were identified:

(a1) the case (a) with ε_1; (a2) the case (a) with ε_2;

(b1) the case (b) with ε_1; (b2) the case (b) with ε_2;

(c1) the case (c) with ε_1; (c2) the case (c) with ε_2.

For each of these six cases, calculating e_λ and e_η , following the formulae (11) and (12) respectively, 12 differential equations follow [3].

The behavior of the expressions (11) and (12) was analyzed and plotted with *Interactive Plot Builder.* Between the 12 plots which have resulted, a few plots were chosen for identifying some special events. The plots were labeled according to the case of simulation.

Fig. 1- the surface deformation in the case a1)

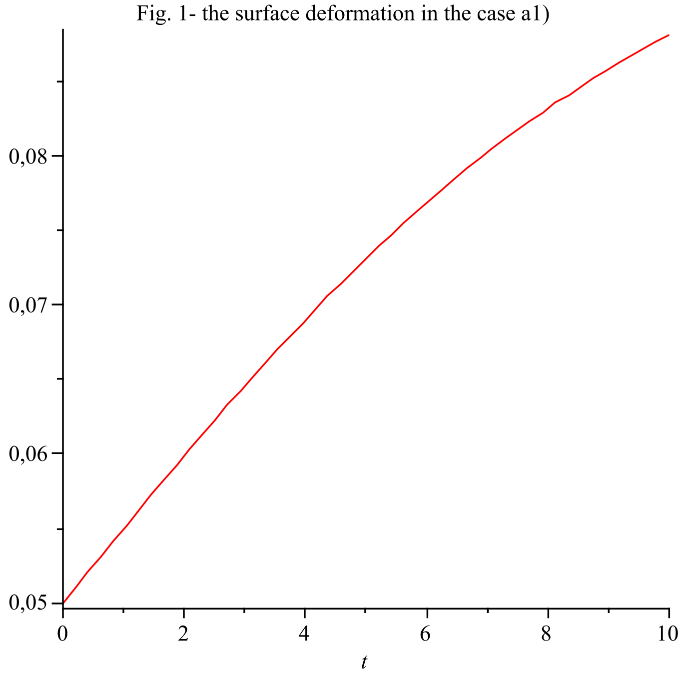


Fig.2 The surface deformation in the case a2)

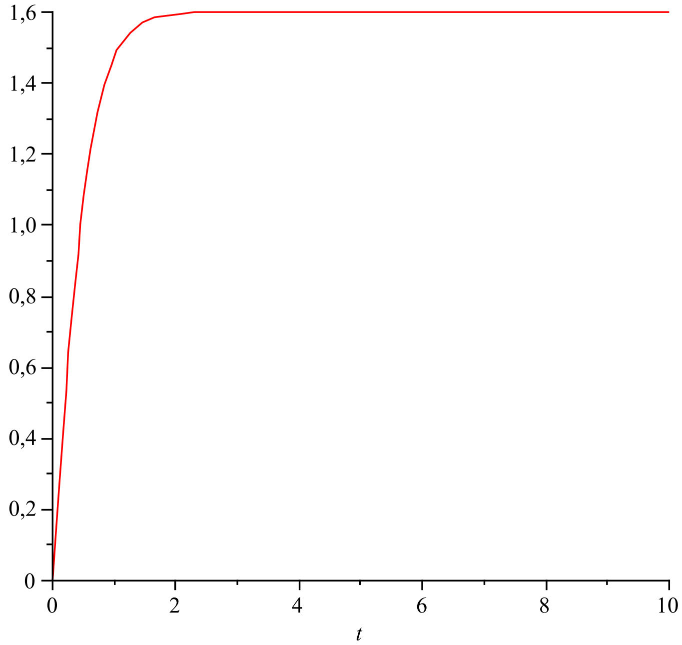


Fig. 3 The length deformation in the case b1)

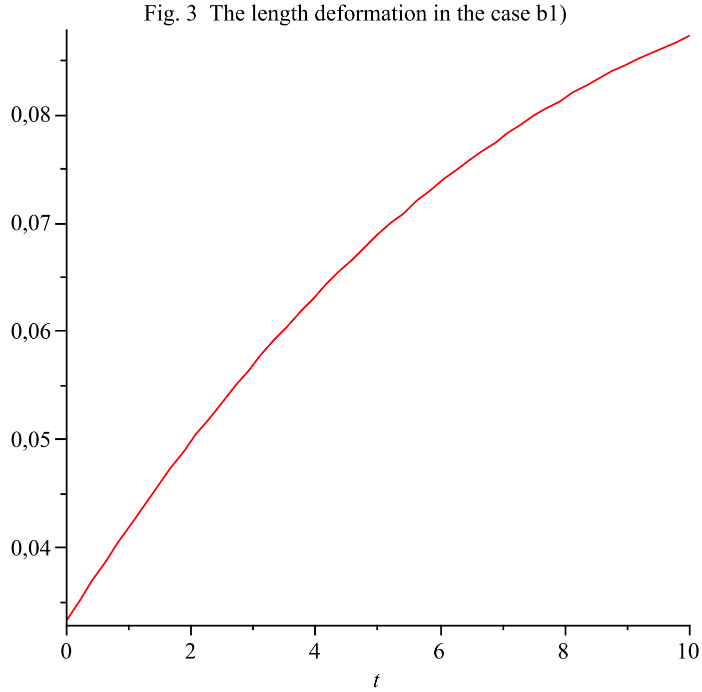


Fig. 4 The surface deformation in the case b1)

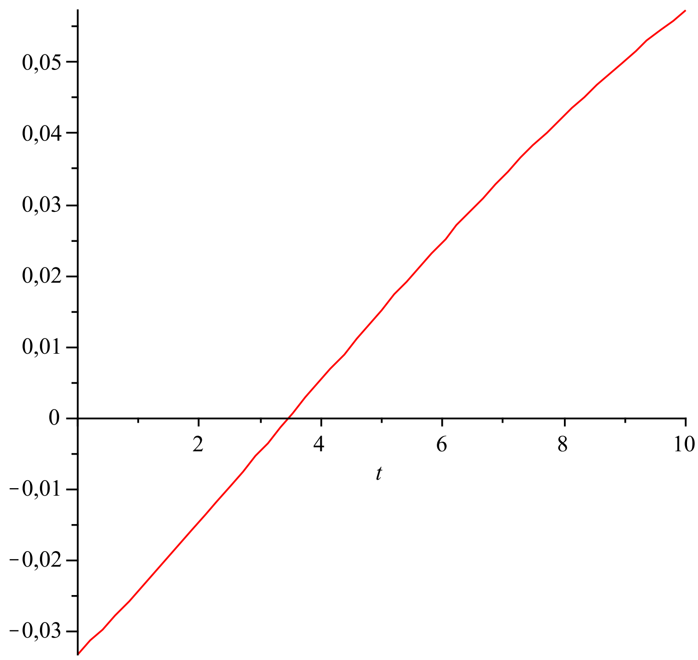


Fig. 5 The surface deformation in the case b2)

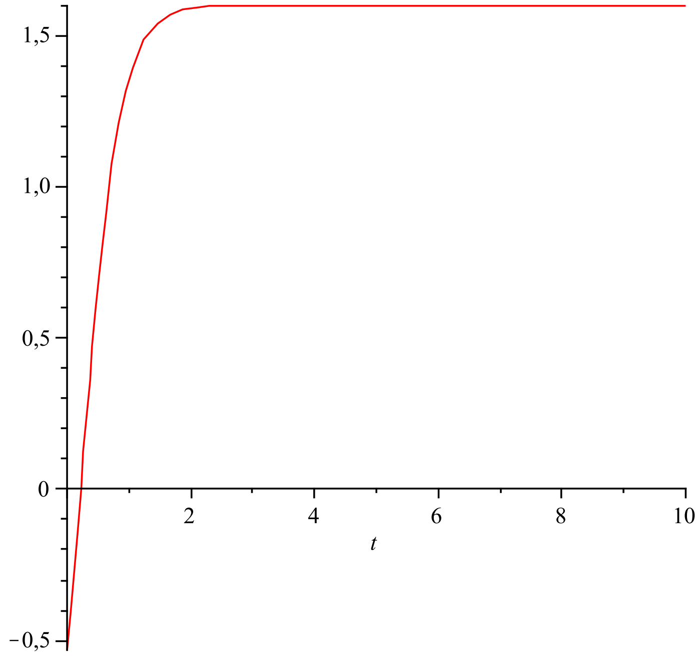
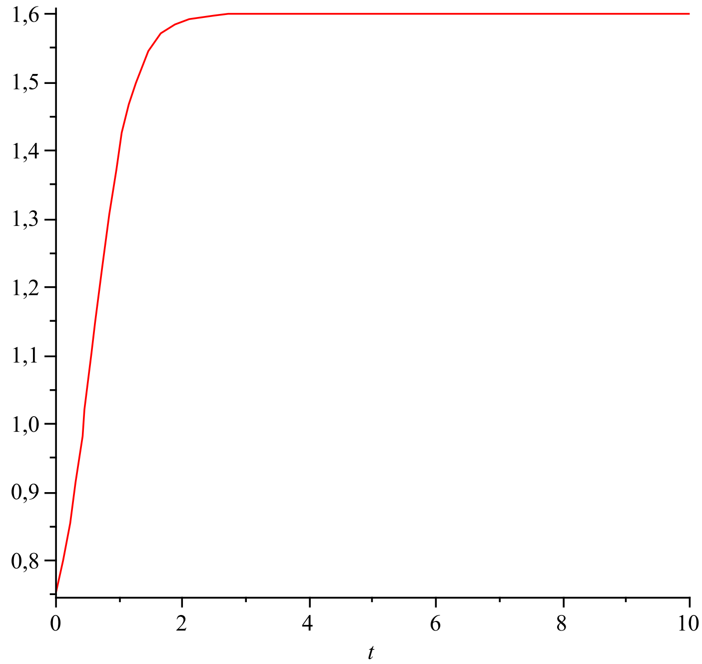
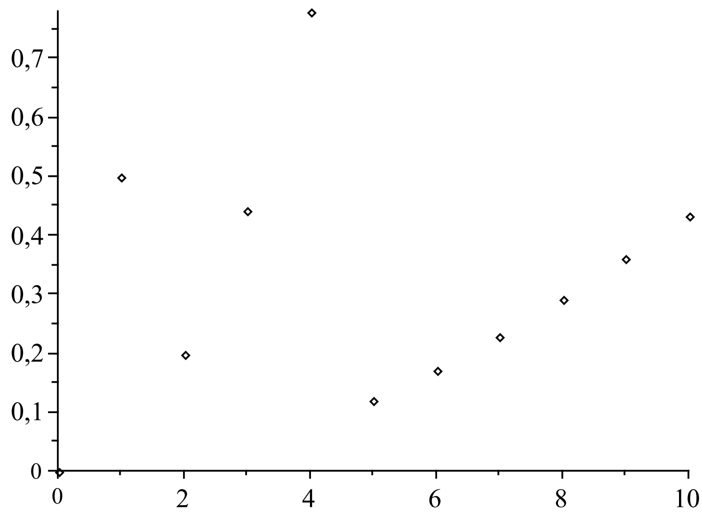


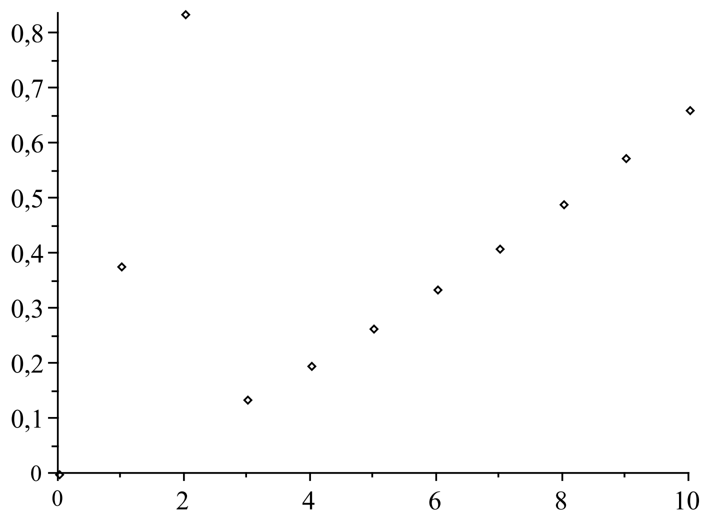
Fig. 6 The surfece deformation in the case c2)



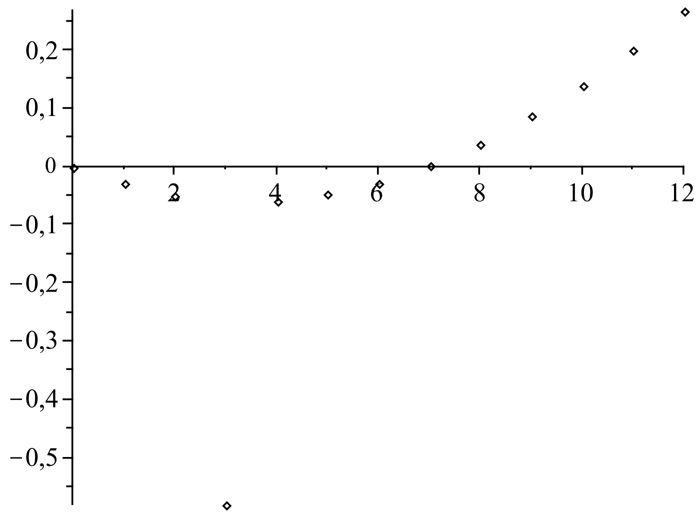
3. CONCLUSIONS. REMARKS

1.Compared to the discrete case, in the continuous time interval case, the behavior of both length and surface deformation seems to be linear. Despite this fact, the deformation is not always linear. It suffices to take into account the cases a1) and b1) for e_λ, b2) for e_η, respectively, as in [3]. The plots are as follows





Looking at the above plots, it seems surprising *that the same expression, with the same parameter values, when studied in discrete time interval produces*



a specific output type, and when studied in continuous time produces another output type.

2. It happens that both the length and surface deformation have a *negative behavior*, although only a small time scale was considered, a 0..10 scale. Thus, comparing to the cases studied in [1], it can be assessed that there can issue significant differences between output data for two plot builders. The Interactive Plot Builder is not more accurate than the discrete plot builder, in spite of the fact that it is faster.

3. It can be assessed, as for 2D periodic case [2], that there *irrational unit vector values* could produce nonlinear phenomena. As an immediate aim, more irrational unit vectors values will be taken into account. That will be useful also for studying the efficiency of deformations, in length and also in surface. As perturbing the initial model, the calculus could become quite complex, therefore a parametric approach would be very useful.

4. The analysis of the length and surface deformation for more irrational values in the periodic case, could match the experiments in [6], as was the situation in 3D case. Moreover, *reiterative events* can be observed. This allows us to take into account the possibility of building some fractal sets. This is a next aim.

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