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A GENERALIZED MATHEMATICAL CAVITATION EROSION MODEL

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Abstract The methods most used to estimate the cavitation erosion resistance pay a special attention to the velocity erosion curve. Depending on the nature and condition of eroded materials, other kind of the volume loss rate curve of erosion cavitations progress is proposed. This model gives a new vision of the volume loss rate curve and generalize some previous mathematical models.

Keywords: erosion, cavitation, loss curve, mathematical model.

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1. INTRODUCTION

The cavities are formed into a liquid when the static pressure of the liquid is reduced below the vapor pressure of the liquid in current temperature. If the cavities are carried to higher-pressure region they implode violently and very high pressures can occur. The cavitation phenomenon may cause serious changes in the microstructure and intrinsic stress level of the material. Macroscopically, the change in hardness is often observed; microscopically, the slip bands and deformation twins appear, and the phase transformations may occur in unstable alloys.

Cavitation erosion is a progressive loss of material from a solid due to the impact action of the collapsing bubbles or cavities in the liquid near the material surface. The models describing the mathematical relations between the volume loss and time in the cavitation erosion was intensely studied but the problem of the analytical description of the characteristic curves for cavitation erosion remained open.

We propose a mathematical model, which permits to define the volume loss curve of material and volume loss rate curve of cavitation erosion.

Usually, the volume loss curve [3], [4], [5], [6] is described by the formula

$$V(t) = A \cdot \Im \cdot U(k,t), \tag{1}$$

or formula

$$V(t) = A \cdot U(k, \Im t), \tag{2}$$

with A-the eroded surface area; $\Im$-measure of cavitations intensity; U- erosion progress function resulting out of applied phenomenological model; k- a set of real parameters (usually 3 parameters are quite sufficient) determined by fitting the erosion curve to the experimental data; t-cumulative exposure duration.

For the volume loss curve V and for volume loss rate curve $v = \frac{dV}{dt}$, for unity eroded surface area, usually [4], [5], [6] the pictures from fig. 1, fig. 2 are proposed.

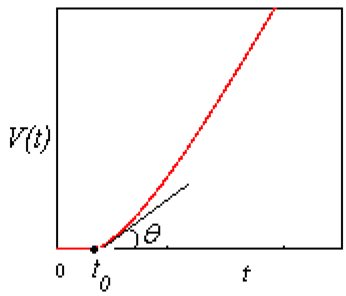

Fig. 1. The volume loss rate curve.

The volume loss rate curve (fig. 2) can be divided into four typical periods:

- incubation period I, is an initial period of damage in which volume loss of material is nearly zero (non-measurable). During the incubation period, a considerable plastic deformation occurs, without any apparent weight loss. In this time interval, the material accumulates energy;

- acceleration period A. In this time interval, the intensification of damage is observed, distinguished by violent increase of volume loss rate of erosion and the volume loss rate reaches the maximal value;

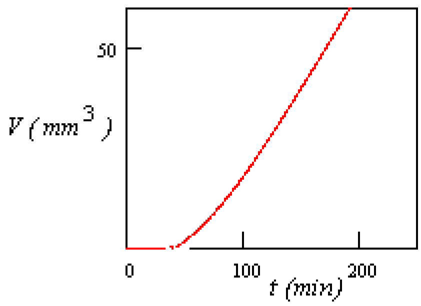

Fig. 2. The volume loss rate curve

- deceleration period D. In this time interval, volume loss rate decreases;

- steady state erosion period S, characterized by almost constant volume loss rate of erosion.

2. A "DAMPED" MODEL OF CAVITATION EROSION

In [1] the volume loss curve is given by formula

$$V(t) = A[v_s t - f(t)], \tag{3}$$

where A is the eroded surface area, v_s is the ultimate value of the volume loss rate and $f(t)$ is the solution of the second-order homogenous linear ordinary differential equations of with constant coefficients

$$\frac{d^2y}{dt^2} + 2\beta\frac{dy}{dt} + \beta^2 = 0 \tag{4}$$

which describes the "damped" oscillations with an "infinite period". Solving the ODE (4) and using (3) it follows that the volume loss is given by formula

$$V(t) = A[v_s t - \lambda t e^{-\beta t}], \tag{5}$$

and the volume loss rate curve is given by formula

$$v(t) = A[v_s - \lambda e^{-\beta t} + \lambda\beta e^{-\beta t}]. \tag{6}$$

The real parameters v_s , λ and β will be determined by fitting the erosion curve to the experimental data (by using the least squares method or another numerical method).

3. A GENERALIZATION OF "DAMPED" MODEL OF CAVITATION EROSION

Remark. The standard curve presented in fig. 2., is acceptable mathematically but it is not very good. During the incubation period, the volume loss V, and the volume loss rate $v = \frac{dV}{dt}$ are null. At the end of the incubation period, the volume loss rate curve $v(t)$ can have a point of discontinuity, because the volume rate curve, may be not smooth at the end point of the incubation period.

Let $(t_0, 0)$ be the end point of the incubation period. If θ is the angle between the time axis and the tangent to the volume loss curve at $(t_0, 0)$, and $\tan\theta \neq 0$ (fig. 3), then we have a discontinuity point for the volume loss rate curve $v(t)$, as in fig. 4.

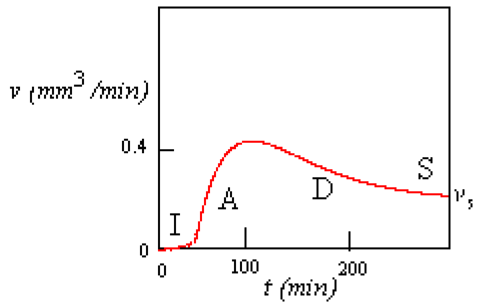

Fig. 3. The volume loss curve near of the end point of the incubation period.

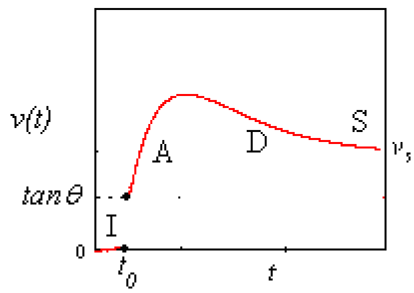

Fig. 4. The volume loss rate curve with a possible discontinuity at the end point of the incubation period.

During the incubation period, the volume loss and the volume loss rate curves are null and the our study is superfluous. In order to simplify the calculations, we choose the time interval $[-\varepsilon, 0]$ as the incubation period and we study the volume loss and the volume loss rate curves for $t \geqslant 0$ ($t_0 = 0$ is the end point of the incubation period)

In most of situations (e.g. in the case of damped vibrations) the volume loss rate curve $v(t)$ can look as in fig. 5 (the incubation period was chosen to be the time interval $[-\varepsilon, 0]$).

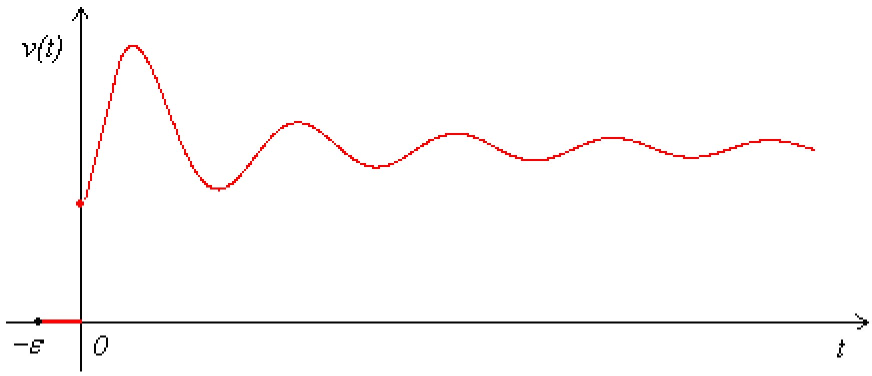

Fig. 5. New model of the volume loss rate curves.

Like in [1], for $v(t)$ we get

$$v(t) = \frac{dV(t)}{dt} = A[v_s t - \frac{df}{dt}(t)],$$

but now, $\frac{df}{dt}$ must satisfy the ODE

$$\frac{d^2y}{dt^2} + \alpha\frac{dy}{dt} + \beta y = 0, \qquad (7)$$

where α and β are real constants, depending on the eroded material; $\alpha \geq 0$, $\beta > 0$. Equation (7) has a physical interpretation like in the case of damped vibrations.

Since the eroded material, tested in Hydraulic Machinery Laboratory by using a vibratory device with a nickel tube, is subject to a frictional force and to a damping force, Newton's second law reads

$$m\frac{d^2y}{dt^2} = damping force + restoring force = -p\frac{dy}{dt} - qy. \qquad (8)$$

Putting in (8) $\alpha = \frac{p}{m}$ and $\beta = \frac{q}{m}$, we obtain the equation (7), which is a second-order linear ordinary differential equation. Its auxiliary equation reads

$$r^2 + \alpha r + \beta = 0. \tag{9}$$

and has the roots $r_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta}}{2}$.

Case I. If $\alpha = 2\sqrt{\beta}$, the roots of equation (9) are $r_1 = r_2 = -\sqrt{\beta}$, and the equation (7) is, like (4), discussed in [1]. Taking into account that $f(0) = 0$, $\lim_{t\to\infty} f(t) = 0$ and $\lim_{t\to\infty} \frac{df(t)}{dt} = 0$ we have the volume loss curve

$$V(t) = A[v_s t - \lambda t e^{-\sqrt{\beta} t}] \tag{10}$$

and the volume loss rate curve is

$$v(t) = A[v_s - \lambda e^{-\sqrt{\beta} t} + \lambda \sqrt{\beta} t e^{-\sqrt{\beta} t}]. \tag{11}$$

Typical graphs of v as a function of t are shown in fig. 4.

Case II. If $\alpha^2 - 4\beta < 0$, then the roots of auxiliary equation (9) are complex: $r_1 = \frac{-\alpha}{2} + \gamma i$, $r_2 = \frac{-\alpha}{2} - \gamma i$, where $\gamma = \sqrt{4\beta - \alpha^2}$.

Since $\frac{df}{dt}$ is the general solution of equation (7) we have:

$$\frac{df}{dt}(t) = e^{\frac{-\alpha}{2} t}(C_1 \cos \gamma t + C_2 \sin \gamma t), \tag{12}$$

and

$$f(t) = \frac{2e^{\frac{-\alpha}{2} t}}{\alpha^2 + 4\gamma^2}(2C_1 \gamma \sin \gamma t - C_2 \alpha \sin \gamma t - 2C_2 \gamma \cos \gamma t - C_1 \alpha \cos \gamma t). \tag{13}$$

Obviously, the conditions: $\lim_{t\to\infty} f(t) = 0$ and $\lim_{t\to\infty} \frac{df(t)}{dt} = 0$ are satisfied. Using formula (13), the condition $f(0) = 0$ implies

$$2C_2 \gamma + C_1 \alpha = 0. \tag{14}$$

Then, using formulae (10) and (14), the volume loss curve is given by

$$V(t) = A\left[v_s t + \frac{2e^{\frac{-\alpha}{2} t}}{\alpha^2 + 4\gamma^2} C_2 \left(\frac{4\gamma^2}{\alpha} \sin \gamma t + \alpha \sin \gamma t\right)\right] \tag{15}$$

and the volume loss rate curve is

$$v(t) = A[v_s - e^{\frac{-\alpha}{2} t} C_2 (\sin \gamma t - \frac{2\gamma}{\alpha} \sin \gamma t). \qquad (16)]$$

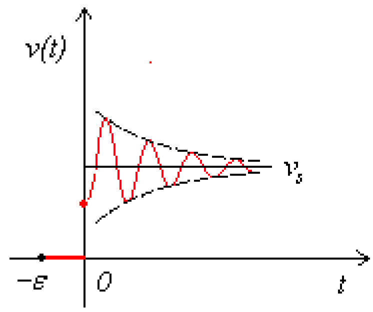

Fig. 6. The volume loss rate curve for case II.

Typical graphs of v as a function of t are shown in fig. 6.

Case III. If $\alpha^2 - 4\beta > 0$, then auxiliary equation (11) has distinct real roots r_1, r_2. Then we have: $\frac{df}{dt}(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$ and $f(t) = \frac{C_1}{r_1} e^{r_1 t} + \frac{C_2}{r_2} e^{r_2 t}$. Since α, β are positive and $\sqrt{\alpha^2 - 4\beta} < \alpha$, the roots r_1, r_2 must both be negative and the conditions $\lim\limits_{t\to\infty} f(t) = 0$ and $\lim\limits_{t\to\infty} \frac{df(t)}{dt} = 0$ are satisfied. The condition $f(0) = 0$ implies $\frac{C_2}{r_2} = -\frac{C_1}{r_1}$. Then, the volume loss curve is

$$V(t) = A(v_s t - \frac{C_1}{r_1} e^{r_1 t} + \frac{C_1}{r_1} e^{r_2 t}) \tag{17}$$

and the volume loss rate curve reads

$$V(t) = A(v_s - C_1 e^{r_1 t} + \frac{C_1 r_2}{r_1} e^{r_2 t}).$$

Typical graphs of v as a function of t are shown in fig. 4.

CONCLUSIONS

The advantage of using ODEs for theoretical analytical erosion curves is that the parameters appear in a natural way in the solution of ODEs and it is often not necessary to decide a priori the number of these parameters. Usually, the real parameters which appear in the expressions of $V(t)$ or $v(t)$ can be determined by fitting the erosion curve to the experimental data, using the least squares method or another numerical method.

Depending on nature and condition of eroded material, we have presented a new possible image of the erosion curves according to the experimental data. Practically, the experimental data may suggest the best mathematical model selection for a given material subject to erosion.

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