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CLASSIFICATION OF THE CUBIC DIFFERENTIAL SYSTEMS WITH SEVEN REAL INVARIANT STRAIGHT LINES

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Abstract A result regarding the classification of the cubic differential systems with seven real invariant straight lines is presnted.

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We consider the cubic differential system

$$\dot{x} = P(x,y),\ \dot{y} = Q(x,y), \tag{1}$$

where P, $Q \in \mathbb{R}[x,y]$, $max\{deg(P), deg(Q)\} = 3$ and $GCD(P,Q) = 1$.

The straight line $Ax+By+C=0$ is said to be invariant for (1) if there exists a polynomial $K(x,y)$ such that the identity $A \cdot P + B \cdot Q \equiv (Ax+By+C) \cdot K$ holds. Let $K(x,y) \equiv (Ax+By+C)^m \cdot K^*(x,y)$, where $m \in \mathbb{N}$, $K^* \in \mathbb{R}[x,y]$ and $Ax+By+C=0$ does not divide $K^*(x,y)$. Then we say that the invariant straight line has the degree of invariance $m+1$.

A set of invariant straight lines can be infinite, finite or empty. In the cases, the number of invariant straight lines is finite, this number is at most eight.

A qualitative investigation of cubic systems with exactly eight and exactly seven invariant straight lines was carried out in [1-3]. In this paper a similar qualitative investigation is done for cubic differential systems with exactly seven real invariant straight lines. It is proved

Theorem. *Any cubic differential system possessing real invariant straight lines with total degree of invariance seven via affine transformation and time rescaling can be written as one of the following seven systems*

$$\begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)(y-a), \\ a > 0,\ a \neq 1; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)((2+a)x-(1+a)y-a), \\ a > 0,\ a \neq 1; \end{cases}$$

$$\begin{cases} \dot{x} = x^3, \\ \dot{y} = y^2(dx+(1-d)y), \\ d(1-d)(d-3)(2d-3) \neq 0; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)((1-a)x+ay-a), \\ a > 0,\ a \neq 1; \end{cases}$$

$$\begin{cases} \dot{x} = x^2(bx+y), \\ \dot{y} = y^2((2+3b)x-(1+2b)y), \\ b(b+1)(2+3b)(1+2b) \neq 0; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(a+(2a-1)x+y), \\ \dot{y} = y(y+1)(a+(3a-1)x+(1-a)y), \\ a(2a-1)(1-a)(3a-1)(3a-2) \neq 0; \end{cases}$$

$$\begin{cases} \dot{x} = x^2(x+1), \\ \dot{y} = y^2(y+1). \end{cases}$$

For the obtained cubic systems the qualitative investigation was performed.

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