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GENERALIZED PRIORITY MODELS WITH "LOOK AHEAD" STRATEGY: NUMERICAL ALGORITHMS FOR BUSY PERIODS

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Abstract Some analytical results concerning busy period distributions for $M_2|G_2|1$ generalized queueing systems with semi-Markov switching and so called "look ahead" strategy are discussed in this paper. We show that the presented analytical results can be viewed as a 2- dimensional analog of the well-known in queueing theory Kendall-Takacs functional equation. Numerical algorithms and modelling examples for busy periods are also presented.

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1. INTRODUCTION

By the Generalized Priority Models (GPM) we understand mathematical models of queueing systems, in which the switching of the service process from a class of requests (messages) to another is non-zero. Such switching between the priority classes is considered to be a random variable with arbitrary distribution function. The GPM can be defined by setting four identifiers: "priority type", "strategy in free state", "discipline of service" and "discipline of switching". As shown in [1] and in some recent publications (see, for example [2] and [3]), GPM have a number of important distinguished features, compared with classical priority models. One of these distinguished features consists in the fact that mathematical formalization of switchover times leads to various new priority laws enabling one to consider more flexible real time processes, such as, absolute, semi-absolute, relative, etc., priority disciplines. Another import feature of GPM consists in the fact that they enable one to consider the strategy of server in the free states.

There are several models of behaviour of the server when the system becomes empty. One of the most studied models is "set to zero" – upon completion of service of the last request in the system the server immediately switches to the neutral state. In the following we analyze a model which is less investigated, namely "look ahead" – the server switches itself to the 1-requests line at the moment when the system becomes empty.

2. DEFINITIONS AND NOTATIONS

Consider the case of "Look Ahead" strategy for priority queueing systems with two priority classes $M_2|G_2|1$.

Consider a queueing system with a single station and 2 classes of incoming requests, each having its own flow of arrival and waiting line. We call the requests from the i^{th} queueing line L_i i-requests. i-Requests have a higher priority than j-requests if $1 \le i < j \le r$. The station gives preference in service to the requests of the highest priority among those presented in the system.

Adopting and slightly extending the standard Kendall notation we write $M_2|G_2|1|\infty$ to denote a priority queueing system with two Poisson incoming flows of requests and random switchover times.

Suppose that the time periods between two consecutive arrivals of the requests of the class i are independent and identically distributed with some common cumulative distribution function (cdf) $A_i(t)$ with mean $\mathbb{E}[A_i]$, $i = 1, 2$. Similarly, suppose that the service time of a customer of the class i is a random variable B_i with a cumulative distribution function $B_i(t)$ with mean service time $\mathbb{E}[B_i]$, $i = 1, 2$.

However, some time is needed for server to proceed with the switching from one line of requests to another. This time is considered to be a random variable, and we say that C_{ij} is the time of switching from the service of i- requests to the service of j-requests, if $1 \le i, j \le r$, $i \ne j$.

We adopt classification and terminology introduced in [1]. We also explain some additional notions and notations.

Definition 2.1. *By a k-busy period we understand the period of time which starts when an i-request enters the empty system, $i \le k$, and finishes when there are no longer k-requests in the system. Denote the k-busy period by Π_k.*

Note, that a 2-busy period is nothing but the system's busy period Π, i.e. $\Pi \equiv \Pi_2$.

Definition 2.2. *By a $\overline{\Pi}_k$ – period we understand the period of time which starts from the moment of arrival of a k-request when there are no i-requests ($i < k$) in the system and ending when the system is free of k-requests.*

Definition 2.3. *By a k-cycle of service we understand the period of time which starts when server begins the servicing of a k-request, and finishes when this request leaves the system. Denote the k-cycle of service by H_k.*

Definition 2.4. *By a k-cycle of switching we understand the period of time which starts when server begins to switch to the line of k-requests, and finishes when server is ready to provide service to these requests. Denote the k-cycle of switching by N_k.*

Let $\overline{\Pi}_k(t)$, $\Pi_k(t)$, $H_k(t)$ and $N_k(t)$ be the cumulative distribution functions of $\overline{\Pi}_k$-busy periods, k-busy periods, k-cycle of service and k-cycle of switching, correspondingly. Let also $\overline{\pi}_k(s)$, $\pi_k(s)$, $h_k(s)$ and $\nu_k(s)$ be their Laplace–Stieltjes transform, i.e.

$$\overline{\pi}_k(s) = \int_0^\infty e^{-st} d\overline{\Pi}_k(t), \ldots, \ \nu_k(s) = \int_0^\infty e^{-st} dN_k(t).$$

Finally, let $\beta_k(s)$ be the Laplace–Stieltjes transform of $B_k(t)$, i.e.

$$\beta_k(s) = \int_0^\infty e^{-st} dB_k(t).$$

Let $C_{12}(t)$ and $C_{21}(t)$ be the cumulative distribution functions of C_{12} and C_{21}. Let also c_{12} and c_{21} be their Laplace–Stieltjes transform, i.e.

$$c_{12}(s) = \int_0^\infty e^{-st} dC_{12}(t),$$

$$c_{21}(s) = \int_0^\infty e^{-st} dC_{21}(t).$$

In this paper we consider next situation for queueing system $M_2|M_2|1$:
for the orientation

- *ON* – non-identical orientation again

  when 1-request arrives in the system the orientation to 2-request is interrupted; after the service of 1-request is finished, the interrupted orientation begins *again non-identical time* of orientation, but this time has the same distribution.

- *OR* – resume interrupted orientation

  when 1-request arrives in the system the orientation to 2-request is interrupted; after the service of 1-request is finished, the interrupted orientation is *resumed.*

- *OC* – orientation is not interrupted, it continues

  when 1-request arrives in the system the orientation to 2-request *is not interrupted.*

on the service

- *SN* – non-identical service again

when 1-request arrives in the system the service of 2-request is interrupted, after the service of 1-request is finished, the interrupted service begins *again non-identical time* of service, but this time has the same distribution.

- SR – resume interrupted service

  when 1-request arrives in system the service to 2-request is interrupted, after the service of 1-request is finished, the interrupted service is *resumed*.

- SL – the request is lost

  when 1-request arrives in system the service of 2-request is interrupted and 2-request is lost.

By combining possible regimes of orientation and service one can obtain 9 types of station operation. Using the above notation, for example ON_SR indicate non-identical orientation again and resume interrupted service.

3. BUSY PERIOD AND ITS EVALUATION

As mentioned above the busy period is the period of time which starts when request enters in the empty system, finishes when there are no requests in the system.

The Laplace–Stieltjes transform of busy period can be determined from following system of functional equations [1]:

$$(a_1 + a_2)\pi_2(s) = a_1\pi_{21}(s) + a_2\pi_{22}(s),$$

$$\pi_{21}(s) = \overline{\pi}_1(s + a_2) + \Big\{\overline{\pi}_1(s + a_2[1 - \overline{\pi}_2(s)]) - \overline{\pi}_1(s + a_2)\Big\}\nu_2\Big(s + a_2[1 - \overline{\pi}_2(s)]\Big)\varphi_1(s),$$

$$\pi_{22}(s) = \nu_2\Big(s + a_2[1 - \overline{\pi}_2(s)]\Big)\varphi_1(s),$$

$$\overline{\pi}_2(s) = h_2(s + a_2[1 - \overline{\pi}_2(s)]), \tag{1}$$

$$\overline{\pi}_1(s) = \beta_1(s + a_1[1 - \overline{\pi}_1(s)]), \tag{2}$$

$$\varphi_1(s) = c_{21}(\xi(s + a_2))\{1 - [c_{21}(\xi(\eta(s)) - c_{21}(\xi(s + a_2))]\nu_2(\eta(s))\}^{-1},$$

$$\xi(s) = s + a_1 - a_1\overline{\pi}_1(s), \quad \eta(s) = s + a_2 - a_2\overline{\pi}_2(s),$$

were for respective case:

"non-identical orientation again" – ON

$$\nu_2(s) = c_{12}(s + a_1)\left\{1 - \frac{a_1}{s + a_1}[1 - c_{12}(s + a_1)]\, c_{21}(s + a_1[1 - \overline{\pi}_1(s)])\overline{\pi}_1(s)\right\}^{-1};$$

"resume interrupted orientation" – OR

$$\nu_2(s) = c_{12}\Big(s + a_1\Big[1 - c_{21}(s + a_1[1 - \overline{\pi}_1(s)])\overline{\pi}_1(s)\Big]\Big);$$

"orientation is not interrupted" – OC

$$\nu_2(s) = c_{12}(s+a_1)\Big\{1 - \Big[c_{12}(s+a_1[1-\overline{\pi}_1(s)]) - c_{12}(s+a_1)\Big]c_{21}(s+a_1[1-\overline{\pi}_1(s)])\Big\}^{-1};$$

"nonidentical repeat again" – SN

$$h_2(s) = \beta_2(s+a_1)\Big\{1 - \frac{a_1}{s+a_1}\left[1-\beta_2(s+a_1)\right] c_{21}(s+a_1[1-\overline{\pi}_1(s)])\overline{\pi}_1(s)\nu_2(s)\Big\}^{-1};$$

"resume" – SR

$$h_2(s) = \beta_2\Big(s + a_1\Big[1 - c_{21}(s+a_1[1-\overline{\pi}_1(s)])\overline{\pi}_1(s)\nu_2(s)\Big]\Big);$$

"loss" – SL

$$h_2(s) = \beta_2(s+a_1) + \frac{a_1}{s+a_1}\Big[1-\beta_2(s+a_1)\Big]c_{21}(s+a_1[1-\overline{\pi}_1(s)])\overline{\pi}_1(s)\nu_2(s);$$

Remark 3.1. *Assume that in the above equations the functions $c_{12}(s)$ and $c_{21}(s)$ are null and the system has only one arrival flow then for this case we obtain next equation:*

$$a_1\pi_1(s) = a_1\overline{\pi}_1(s) = a_1\beta_1(s+a_1[1-\overline{\pi}_1(s)]).$$

In this case $\pi(s) = \overline{\pi}_1(s)$ and

$$\pi_1(s) = \beta_1(s+a_1[1-\pi_1(s)])$$

This equation is known as the classical Kendall–Takacs equation.

3.1. ALGORITHMS FOR EVALUATION LAPLACE-STIELTJES TRANSFORM OF BUSY PERIOD.

As can be seen from equations (1) and (2) the function values $\overline{\pi}_2(s)$ and $\overline{\pi}_1(s)$ can be determined using numerical methods only. Thus, to evaluate the Laplace–Stieltjes transform of the busy period and the Laplace–Stieltjes transforms $\nu_2(s)$ and $h_2(s)$ one needs to use numerical algorithms. An efficient method for doing so elaborated in [4] is used in the following algorithms. In the following two algorithms for two different situations are presented. The other situation mentioned in the first section can be treated in a similar way.

Algorithm 1 ($M_2|G_2|1$ ON_SN)

Input: r, $\{a_i\}_{i=1}^2$, $\{\beta_i(s)\}_{i=1}^2$, $c_{12}(s)$, $c_{21}(s)$, ε;

Output: $\pi_2(s^*)$, $h_2(s^*)$, $\nu_2(s^*)$;

Description:

$\sigma := a_1 + a_2;$

$$\pi_2(s^*) = \frac{a_1\pi_{21}(s^*)}{\sigma} + \frac{a_2\pi_{22}(s^*)}{\sigma},$$

$$\pi_{21}(s^*) = \overline{\pi}_1(s^*+a_2)+\Big\{\overline{\pi}_1(s^*+a_2[1-\overline{\pi}_2(s^*)])-\overline{\pi}_1(s^*+a_2)\Big\}\nu_2\Big(s^*+a_2[1-\overline{\pi}_2(s^*)]\Big)\varphi_1(s^*),$$

$$\pi_{22}(s) = \nu_2\Big(s + a_2[1 - \overline{\pi}_2(s)]\Big)\varphi_1(s),$$

$$\varphi_1(s^*) = c_{21}(\xi(s^* + a_2))\{1 - [c_{21}(\xi(\eta(s^*)) - c_{21}(\xi(s^* + a_2))]\nu_2(\eta(s^*))\}^{-1},$$

$$\xi(s^*) = s^* + a_1 - a_1\overline{\pi}_1(s^*),$$

$$\eta(s^*) = s^* + a_2 - a_2\overline{\pi}_2(s^*),$$

$$\nu_2(s^*) = c_{12}(s^* + a_1)\left\{1 - \frac{a_1}{s^* + a_1}\left[1 - c_{12}(s^* + a_1)\right] c_{21}(s^* + a_1[1 - \overline{\pi}_1(s^*)])\overline{\pi}_1(s^*)\right\}^{-1};$$

$$h_2(s^*) = \beta_2(s^*+a_1)\left\{1-\frac{a_1}{s^* + a_1}\left[1 - \beta_2(s^* + a_1)\right] c_{21}(s^*+a_1[1-\overline{\pi}_1(s^*)])\overline{\pi}_1(s^*)\nu_2(s^*)\right\}^{-1};$$

$n := 1; \underset{\sim}{\overline{\pi}}{}_1^{(n)}(0) := 0; \widetilde{\overline{\pi}}{}_1^{(n)}(0) = 1;$

Repeat

$\widetilde{\overline{\pi}}{}_1^{(n)}(s^*) = \beta_1(s^* + a_1[1 - \widetilde{\overline{\pi}}{}_1^{(n-1)}(s^*));$

$\underset{\sim}{\overline{\pi}}{}_1^{(n)}(s^*) = \beta_1(s^* + a_1[1 - \underset{\sim}{\overline{\pi}}{}_1^{(n-1)}(s^*));$

inc(n);

Until $\dfrac{\widetilde{\overline{\pi}}{}_1^{(n)}(s^*) - \underset{\sim}{\overline{\pi}}{}_1^{(n-1)}(s^*)}{2} < \varepsilon;$

$$\overline{\pi}_1(s^*) := \frac{\widetilde{\overline{\pi}}{}_1^{(n)}(s^*) + \underset{\sim}{\overline{\pi}}{}_1^{(n-1)}(s^*)}{2};$$

$n := 1; \underset{\sim}{\overline{\pi}}{}_2^{(n)}(0) := 0; \widetilde{\overline{\pi}}{}_2^{(n)}(0) = 1;$

Repeat

$\widetilde{\overline{\pi}}{}_2^{(n)}(s^*) = h_2(s^* + a_1[1 - \widetilde{\overline{\pi}}{}_2^{(n-1)}(s^*));$

$\underset{\sim}{\overline{\pi}}{}_2^{(n)}(s^*) = h_2(s^* + a_1[1 - \underset{\sim}{\overline{\pi}}{}_2^{(n-1)}(s^*));$

inc(n);

Until $\dfrac{\widetilde{\overline{\pi}}{}_2^{(n)}(s^*) - \underset{\sim}{\overline{\pi}}{}_2^{(n-1)}(s^*)}{2} < \varepsilon;$

$$\overline{\pi}_2(s^*) := \frac{\widetilde{\overline{\pi}}{}_2^{(n)}(s^*) + \underset{\sim}{\overline{\pi}}{}_2^{(n-1)}(s^*)}{2};$$

End of Algorithm 1

Algorithm 2 ($M_2|G_2|1$ $OC_{-}SL$)

Input: r, $\{a_i\}_{i=1}^2$, $\{\beta_i(s)\}_{i=1}^2$, $c_{12}(s)$, $c_{21}(s)$, ε;

Output: $\pi_2(s^*)$, $h_2(s^*)$, $\nu_2(s^*)$;

Description:

$\sigma := a_1 + a_2$;

$$\pi_2(s^*) = \frac{a_1\pi_{21}(s^*)}{\sigma} + \frac{a_2\pi_{22}(s^*)}{\sigma},$$

$$\pi_{21}(s^*) = \overline{\pi}_1(s^*+a_2)+\Big\{\overline{\pi}_1(s^*+a_2[1-\overline{\pi}_2(s^*)])-\overline{\pi}_1(s^*+a_2)\Big\}\nu_2\Big(s^*+a_2[1-\overline{\pi}_2(s^*)]\Big)\varphi_1(s^*),$$

$$\pi_{22}(s) = \nu_2\Big(s + a_2[1 - \overline{\pi}_2(s)]\Big)\varphi_1(s),$$

$$\varphi_1(s^*) = c_{21}(\xi(s^* + a_2))\{1 - [c_{21}(\xi(\eta(s^*)) - c_{21}(\xi(s^* + a_2))]\nu_2(\eta(s^*))\}^{-1},$$

$$\xi(s^*) = s^* + a_1 - a_1\overline{\pi}_1(s^*),$$

$$\eta(s^*) = s^* + a_2 - a_2\overline{\pi}_2(s^*),$$

$$\nu_2(s) = c_{12}(s^*+a_1)\Big\{1-\Big[c_{12}(s^*+a_1[1-\overline{\pi}_1(s^*)])-c_{12}(s^*+a_1)\Big]c_{21}(s^*+a_1[1-\overline{\pi}_1(s^*)])\Big\}^{-1};$$

$$h_2(s^*) = \beta_2(s^* + a_1) + \frac{a_1}{s^* + a_1}\Big[1 - \beta_2(s^* + a_1)\Big]c_{21}(s^* + a_1[1 - \overline{\pi}_1(s^*)])\overline{\pi}_1(s^*)\nu_2(s^*);$$

$n := 1$; $\underset{\sim}{\overline{\pi}}_1^{(n)}(0) := 0$; $\overset{\approx}{\pi}_1^{(n)}(0) = 1$;

Repeat

$\overset{\approx}{\pi}_1^{(n)}(s^*) = \beta_1(s^* + a_1[1 - \overset{\approx}{\pi}_1^{(n-1)}(s^*))$;

$\underset{\sim}{\overline{\pi}}_1^{(n)}(s^*) = \beta_1(s^* + a_1[1 - \underset{\sim}{\overline{\pi}}_1^{(n-1)}(s^*))$;

$inc(n)$;

Until $\dfrac{\overset{\approx}{\pi}_1^{(n)}(s^*) - \underset{\sim}{\overline{\pi}}_1^{(n-1)}(s^*)}{2} < \varepsilon$;

$\overline{\pi}_1(s^*) := \dfrac{\overset{\approx}{\pi}_1^{(n)}(s^*) + \underset{\sim}{\overline{\pi}}_1^{(n-1)}(s^*)}{2}$;

$n := 1$; $\underset{\sim}{\overline{\pi}}_2^{(n)}(0) := 0$; $\overset{\approx}{\pi}_2^{(n)}(0) = 1$;

Repeat

$\overset{\approx}{\pi}_2^{(n)}(s^*) = h_2(s^* + a_1[1 - \overset{\approx}{\pi}_2^{(n-1)}(s^*))$;

$\underset{\sim}{\overline{\pi}}_2^{(n)}(s^*) = h_2(s^* + a_1[1 - \underset{\sim}{\overline{\pi}}_2^{(n-1)}(s^*))$;

$inc(n)$;

Until $\dfrac{\overset{\approx}{\pi}_2^{(n)}(s^*) - \underset{\sim}{\overline{\pi}}_2^{(n-1)}(s^*)}{2} < \varepsilon$;

$$\overline{\pi}_2(s^*) := \frac{\overline{\overline{\pi}}_2^{(n)}(s^*) + \underline{\overline{\pi}}_2^{(n-1)}(s^*)}{2};$$

End of Algorithm 2

Remark 3.2. *One can efficiently evaluate the Laplace–Stieltjes transform of busy period using the algorithms described. In order to determine the value of the busy period one needs to use inversion algorithms (for example see [4]).*

3.2. EXAMPLES OF EVALUATIONS OF THE BUSY PERIOD

Example 3.1. *Consider the system $M_2|M_2|1$ (ON_-SN) with interarrival times being distributed exponentially Exp(a_k) $k = 1,2$ and exponential service times Exp(b_k), $b_k = 100$, $k = 1,2$. The switchover times C_k are all distributed exponentially $Exp(\omega)$, $\omega = \omega_{12} = \omega_{21} = 100$. The quantity ε was taken to be* 0.000001.

$$\beta_k(s) = \frac{b_k}{s+b_k}, \quad k = 1,2$$

$$c_{12}(s) = \frac{\omega_{12}}{\omega_{12}+s}, \quad c_{21}(s) = \frac{\omega_{21}}{\omega_{21}+s}.$$

a_k	$\pi_2(0)$	$\nu_2(0)$	$h_2(0)$
10	0.999999	1.000000	1.000000
50	0.545809	0.999999	0.999999
80	0.295837	0.999999	0.999998

Table 1 Calculation of the $\pi_2(0)$, $\nu_2(0)$ and $h_2(0)$

Example 3.2. *Consider the system $M_2|M_2|1$ (ON_-SR) with interarrival times being distributed exponentially Exp(a_k), $a_k = 10$, $k = 1,2$ and service times being distributed according to Erlang law $Er(3, b_k)$, $k = 1,2$. The switchover times C_k are all distributed exponentially $Exp(\omega)$, $\omega = \omega_{12} = \omega_{21} = 200$. The quantity ε was taken to be* 0.000001.

$$\beta_k(s) = \Big(\frac{b_k}{s+b_k}\Big)^3, \quad k = 1,2; \quad c_{12}(s) = \frac{\omega_{12}}{\omega_{12}+s}, \quad c_{21}(s) = \frac{\omega_{21}}{\omega_{21}+s}.$$

Example 3.3. *Consider the system $M_2|M_2|1$ (OR_-SN) with interarrival times being distributed exponentially Exp(a_k) $k = 1,2$ and Erlang service times $Er(2, b_k)$, $b_k =$*

b_k	$\pi_2(0)$	$\nu_2(0)$	$h_2(0)$
35	0.385793	1.000000	0.999999
50	0.689388	1.000000	1.000000
70	0.999996	1.000000	1.000000

Table 2 Calculation of the $\pi_2(0)$, $\nu_2(0)$ and $h_2(0)$

200, $k = 1, 2$. *The switchover times* C_k *are all distributed exponentially* $Exp(\omega)$, $\omega = \omega_{12} = \omega_{21} = 100$. *The quantity* ε *was taken to be* 0.000001.

$$\beta_k(s) = \Big(\frac{b_k}{s + b_k}\Big)^2, \quad k = 1, 2; \quad c_{12}(s) = \frac{\omega_{12}}{\omega_{12} + s}, \quad c_{21}(s) = \frac{\omega_{21}}{\omega_{21} + s}.$$

a_k	$\pi_2(0.5)$	$\nu_2(0.5)$	$h_2(0.5)$
1	0.989696	0.994926	0.994857
10	0.984776	0.993935	0.993169
50	0.469865	0.985422	0.975887

Table 3 Calculation of the $\pi_2(0.5)$, $\nu_2(0.5)$ and $h_2(0.5)$

Example 3.4. *Consider the system* $M_2|M_2|1$ *(*$OC_{-}SL$*) with interarrival times being distributed exponentially Exp(*a_k*),* $a_1 = 70$, $a_1 = 1$, $k = 1, 2$ *and exponential service times Exp(*b_k*),* $b_k = 100$, $k = 1, 2$. *The switchover times* C_k *are all distributed according Gamma* $Ga(2.5; \omega)$ *law,* $\omega = \omega_{12} = \omega_{21}$, $\omega = 200$. *The quantity* ε *was taken to be* 0.000001.

$$\beta_k(s) = \frac{b_k}{s + b_k}, \quad k = 1, 2;$$

$$c_{12}(s) = \Big(\frac{\omega}{\omega + s}\Big)^{2.5},$$

$$c_{21}(s) = \Big(\frac{\omega}{\omega + s}\Big)^{2.5}.$$

s	$\pi_2(s)$	$\nu_2(s)$	$h_2(s)$
0	0.999999	0.999999	0.999999
0.5	0.980521	0.963790	0.968226
1.0	0.963587	0.932315	0.940896
5.0	0.870813	0.768586	0.803302

Table 4 Calculation of the $\pi_2(s)$, $\nu_2(s)$ and $h_2(s)$

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