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# A CRITERION FOR PARAMETRICAL COMPLETENESS IN THE 5-VALUED NON-LINEAR ALGEBRAIC MODEL OF INTUITIONISTIC LOGIC

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**Abstract** A.V. Kuznetsov [2, p.28] put the problem to find out conditions for parametrical completeness of any system of formulas in the Intuitionistic Propositional Logic. In the present paper we solve a more weak problem. We find out conditions permitting to determine the parametrical completeness in the logic of 5-valued non-linear pseudo-boolean algebra. We give the suitable solution in terms of 11 parametrical pre-complete classes of formulas.

**Keywords:** Intuitionistic Logic, parametrical expressibility, parametrical completeness, pre-complete system, pseudo-boolean algebra.
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## 1. INTRODUCTION

A.V. Kuznetsov [2] introduced the concept of parametrical expressibility and considered the problem of parametrical completeness in the Intuitionistic Logic ([2, p.28], problem 16). For solving this problem it is natural to find out conditions for parametrical completeness in some intermediate more simple logics that approximate the Intuitionistic logic.

In the present paper we establish the necessary and sufficient conditions for the parametrical completeness of an arbitrary system of formulas in the logic of 5-valued pseudo-boolean algebra with two incomparable elements.

Formulas (of propositional logic) are constructed from variables $p, q, r$ by means of logical operations: & (conjunction), $\vee$ (disjunction), $\supset$ (implication), $\neg$ (negation). Using the mark $\leftrightharpoons$, and reading it as "means" we introduce some designations for the formulas: $1 \leftrightharpoons (p \supset p)$, $0 \leftrightharpoons (p \,\&\, \neg p)$, $(F \backsim G) \leftrightharpoons ((F \supset G)\&(G \supset F))$ (equivalence).

The Intuitionistic and classical propositional calculi are based on the mentioned notion of formula. We determine the logic of some calculus as the set of all formulas deducible in considered calculus.

It is known that the pseudo-boolean algebras [3] represent the algebraic interpretations of intuitionistic logic. Let consider the 5-valued non-linear pseudo-boolean algebra $Z_5 \ = <\ \{0, \rho, \sigma, \omega, 1\};\ \&\, , \vee\, , \supset, \neg\ >$ where $0 < \rho < \omega < 1$, $0 < \sigma < \omega$, $\rho \& \sigma = 0$, $\rho \vee \sigma = \omega$. It is clear that the algebra $Z_3 = \ < \{0, \omega, 1\};\ \&\, , \vee,\ \supset,\ \neg >$ is a subalgebra of $Z_5$.

We define the logic $LZ_5$ of the algebra $Z_5$ as the set of all formulas true on $Z_5$, i.e. the set of formulas identically equal to the greatest element 1 of this algebra.

A $F$ formula is called directly expressible via the $\Sigma$ system of formulas if it is possible to obtain $F$ from variables and formulas of $\Sigma$ using a finite number of weak substitution rule. A formula $F$ is said to be parametrically expressible ($p$. expressible) in a $L$ logic in terms of a $\Sigma$ system of formulas, if there exist numbers $l$ and $s$, variables $\pi, \pi_1, \ldots, \pi_l$ not occurring in $F$, pairs of formulas $A_i, B_i$ $(i = \overline{1, s})$ directly expressible in $L$ in terms of $\Sigma$ and formulas $D_1, \ldots, D_l$, not containing the variables $\pi, \pi_1, \ldots, \pi_l$, such that the following relations take place:

$$L \vdash ((F \sim \pi) \sim (A_1 \sim B_1)\,\&\, \ldots\, \&\,(A_s \sim B_s)[\pi_1 \,|\, D_1], \ldots, [\pi_l \,|\, D_l])$$

$$L \vdash ((A_1 \sim B_1)\,\&\, \ldots\, \&\,(A_s \sim B_s) \supset (F \sim \pi)).$$

A $\Sigma$ system of formulas is said to be parametrically complete ($p$. complete) in a $L$ logic if all formulas of $L$ language are $p$. expressibly in $L$ in terms of $\Sigma$.

One says that a formula $F(p_1, \ldots, p_n)$ preserves the predicate $R(x_1, \ldots, x_m)$ on algebra $A$ if, for any elements $\alpha_{ij} \in A$ $(i = \overline{1, m};\ j = \overline{1, n})$, the truth of propositions

$$R[\alpha_{11}, \alpha_{21}, \ldots, \alpha_{m1}], \ldots, R[\alpha_{1n}, \alpha_{2n}, \ldots, \alpha_{mn}]$$

implies

$$R[F[\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1n}], \ldots, F[\alpha_{m1}, \alpha_{m2}, \ldots, \alpha_{mn}]].$$

Let consider the following predicate on $Z_5$ algebra: $g(x) = y$, where $g(0) = 0$; $g(\rho) = \sigma$; $g(\sigma) = \rho$; $g(\omega) = g(1) = 1$.

**Theorem.** *In order that a system $\Sigma$ of formulas be p. complete in the logic $LZ_5$ it is necessary and sufficient that $\Sigma$ be p. complete in the logic $LZ_3$ and there exists a formula $F$ of $\Sigma$ which does not preserve the predicate $g(x) = y$ on the algebra $Z_5$.*

## References

[1] Cucu I.V., Raţă M.F. *Parametrical completeness in the logic of simplest pseudo-boolean algebra with two incomparable elements*, Bul. Acad. of Sci. of Moldova, 1(7)(1992), 46–51 (in Russian).

[2] Kuznetsov A.V. *On tools for discovery of nondeducibility or nonexpressibility*, Logical Deduction, Moscow, Nauka, 1979, 5-33 (in Russian).

[3] Rasiova H., Sikorski R., *The mathematics of Metamathematics*, Warsawa, (1963).