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PROPAGATION OF TRANSVERSE AND MICROROTATIONAL WAVES IN MICROPOLAR GENERALIZED THERMODIFFUSION ELASTIC HALF SPACE

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Abstract This study deals with the propagation of transverse and microrotational waves in micropolar thermodiffusion elastic half-space in context of non classical (generalized) theories of thermoelasticity developed by Lord and Shulman (L-S theory) and Green and Lindsay (G-L theory). The amplitude ratios are obtained in closed form and numerically simulated results are presented graphically against angle of incidence. The phenomenon of micropolarity, diffusion and impact of relaxation times are studied on the amplitude ratios. The results of some earlier investigations have also been deduced as particular case from the present investigation.

Keywords: micropolar, thermodiffusive, amplitude ratios, reflection, transverse and microrotational waves.

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1. INTRODUCTION

In recent years, micropolar theory gained great importance due to discrepancies observed in classical theory and experimental results while studying the stress concentration in the neighborhood of the holes and rocks, especially in the material containing laminates, granular fibers. Micropolar elastic material solid differs from a classical elastic material in that each point has extra rotational degrees of freedom independent of translation and that the material can transmit couple stress as well as usual force stress. Also modern engineering structures are often made up of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse structure come in this category. Classical elasticity is inadequate to represent the behavior of such materials. The analysis of such materials requires incorporating the theory of oriented media.

For this reason, micropolar theories are developed by Eringen [1, 2] for elastic solids, fluid and further for non-local polar fields. Also Nowacki [3] developed a theory of micropolar coupled thermoelasticity. Touchert et. al [4] derived the basic equations

of linear theory of micropolar coupled thermoelasticity. Dost and Tabarrok [5] presented the micropolar generalized thermoelasticity by using Green Lindsay theory. Chandrasekharaiah [6] formulated a theory of micropolar thermoelasticity which includes heat flux among the constitutive variables. Boschi and Ieşan [7] extended a generalized theory of micropolar thermoelastic media that permits the transmission of heat as thermal waves at finite speed.

Diffusion can be defined as the movement of particles from an area of high concentration to an area of lower concentration until equilibrium is reached. It occurs as a result of second law of thermodynamics which states that the entropy or disorder of any system must always increase with time. Diffusion is important in many life processes and is of great interest due to its various applications in geophysics and industry. Also these days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from deposits. Thermoelastic diffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Nowacki [8, 9, 10, 11] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves.

Gawinecki and Szymaniec [12] proved a theorem about global existence of the solution for a non-linear parabolic thermoelastic diffusion problem. Sherief et. al [13] developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Sherief and Shaleh [14] investigated a half space problem in the theory of generalized thermoelastic diffusion with one relaxation time. Singh [15, 16] investigated the reflection of P and SV waves from free surface of an elastic solid on generalized thermodiffusion elastic medium. Aouadi [17, 18, 19, 20, 21] investigated the different types of problems in thermoelastic diffusion. Sharma et.al [22, 23], Kumar and Kansal [24], Sherief and El Maghraby [25] discussed various types of problems in thermoelastic diffusion. Recently Kumar et. al [26, 27] discussed source problems in micropolar thermodiffusive medium.

In the present paper, reflection from free plane boundary in micropolar generalized thermodiffusion medium is considered to analyze the effect of micropolarity and thermodiffusion on the amplitude ratio's of various reflected waves for a particular model in the context of generalized theories of thermoelasticity. Some special cases of micropolarity and thermodiffusion is also deduced from present investigation.

2. BASIC EQUATIONS

Following Eringen [1], Nowacki [8], Lord and Shulman [28] and Green and Lindsay [29], the governing equations for isotropic, homogeneous micropolar thermodiffusion elastic solid in absence of body forces, body couples, heat sources and diffusive mass sources are the following.

The constitutive relations:

$$t_{kl} = \lambda u_{r,r}\delta_{kl}+\mu(u_{k,l}+u_{l,k})+K(u_{l,k}-\epsilon_{klm}\phi_m)-\beta_1(1+\tau_1\frac{\partial}{\partial t})T\delta_{kl}-\beta_2(1+\tau^1\frac{\partial}{\partial t})C\delta_{kl}, \quad (1)$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}, \quad (2)$$

$$P = -\beta_2 e_{jj} + b(1 + \tau^1\frac{\partial}{\partial t})C - a(1 + \tau_1\frac{\partial}{\partial t})T; \quad (3)$$

Stress equations of motion:

$$(\lambda+\mu)u_{l,kl} + (\mu + K)u_{k,ll} + K\epsilon_{klm}\phi_{m,l} - \beta_1(1 + \tau_1\frac{\partial}{\partial t})T_{,k} - \beta_2(1 + \tau^1\frac{\partial}{\partial t})C_{,k} = \rho\ddot{u_k}; \quad (4)$$

Couple stress equations of motion:

$$(\alpha + \beta)\phi_{l,lk} + \gamma\phi_{k,ll} + K\epsilon_{klm}u_{m,l} - 2K\phi_k = \rho j\ddot{\phi_k}; \quad (5)$$

Equation of heat conduction:

$$\rho C_E(1 + \tau_0\frac{\partial}{\partial t})\frac{\partial T}{\partial t} + \beta_1 T_0(1 + \Omega\tau_0\frac{\partial}{\partial t})\frac{\partial e}{\partial t} + aT_0(1 + \gamma\frac{\partial}{\partial t})\frac{\partial C}{\partial t} = K^* T_{,ii}; \quad (6)$$

Equation of mass diffusion:

$$D\beta_2 e_{,ii} + Da(1 + \tau_1\frac{\partial}{\partial t})T_{,ii} + (1 + \Omega\tau^0\frac{\partial}{\partial t})\frac{\partial C}{\partial t} - Db(1 + \tau^1\frac{\partial}{\partial t})C_{,ii} = 0, \quad (7)$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (i = 1, 2, 3)$$

$$\beta_1 = (3\lambda + 2\mu + K)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu + K)\alpha_c,$$

λ, μ- Lame's constants, α_t- coefficient of linear thermal expansion, α_c - coefficient of diffusion expansion, ρ- density, K^*- thermal conductivity, C_E- specific heat, t_{ij} - components of stress tensor, m_{ij}-components of couple stress tensor, e_{ij}- components of strain tensor, $e = e_{kk}$, δ_{ij}-Kronecker delta, u_i- displacement components, ϕ_i- microrotational components, C-concentration, j- microrotation interia, $K, \alpha, \beta, \gamma, a, b$-material constant, t-time, T-absolute temperature, T_0-temperature of medium in its natural state assumed to be such that $|\frac{T}{T_0}| < 1$, D-thermoelastic diffusion constants, P-chemical potential per unit mass.

$$\tau_1 = \tau^1 = 0, \quad \Omega = 1, \quad \gamma = \tau_0 \qquad \text{for L-S model;}$$
$$\tau_1 > 0, \ \tau^1 > 0, \quad \Omega = 0, \quad \gamma = \tau^0 \qquad \text{for G-L model.}$$

The thermal relaxation times for G-L model satisfies the inequality, $\tau_1 \geq \tau_0 \geq 0$ and diffusion relaxation time satisfies the inequality $\tau^1 \geq \tau^0 \geq 0$.

3. FORMULATION AND SOLUTION OF THE PROBLEM

We consider a homogeneous, isotropic micropolar generalized thermodiffusion elastic solid in undeformed state at temperature T_0, which we designate as the plane $x_3 \geq 0$ of rectangular cartesian co-ordinate $Ox_1x_2x_3$. We consider thermoelastic plane wave in x_1x_3-plane with wave front parallel to x_2-axis and all the fields variables depends only on x_1, x_3 and t.

For two dimensional problem, we take

$$\overrightarrow{u} = (u_1, 0, u_3), \qquad \overrightarrow{\phi} = (0, \phi_2, 0). \tag{8}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{\omega_1}{c_1} x_1, \quad x_3' = \frac{\omega_1}{c_1} x_3, \quad u_1' = \frac{\rho c_1 \omega_1}{\beta_1 T_0} u_1, \quad u_3' = \frac{\rho c_1 \omega_1}{\beta_1 T_0} u_3, \quad t_{33}' = \frac{t_{33}}{\beta_1 T_0},$$

$$t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad m_{ij}' = \frac{\omega_1}{c_1 \beta_1 T_0} m_{ij} \quad C' = \frac{\beta_2}{\rho C_1^2} C, \quad T' = \frac{\beta_1}{\rho C_1^2} T, \quad \phi_2' = \frac{\rho c_1^2}{\beta_1 T_0} \phi_2,$$

$$\tau_o' = \omega_1 \tau_o, \quad \tau_1' = \omega_1 \tau_1, \quad \tau^{o'} = \omega_1 \tau^o, \quad \tau^{1'} = \omega_1 \tau^1, \quad t' = \omega_1 t, \tag{9}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho} \quad \text{and} \quad \omega_1 = \frac{\rho C_E c_1^2}{K^*}.$$

The expression relating displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ to the scalar potential functions $q(x_1, x_3, s)$ and $\psi(x_1, x_3, s)$ in dimensionless form are given by

$$u_1 = \frac{\partial q}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial q}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \tag{10}$$

Making use of equation (8)-(10) in equations (4)- (7) (suppressing the primes for convenience), we obtain

$$((a_1 + a_2)\nabla^2 - \frac{\partial^2}{\partial t^2})q - a_4(1 + \tau_1 \frac{\partial}{\partial t})T - a_4(1 + \tau^1 \frac{\partial}{\partial t})C = 0, \tag{11}$$

$$a_8(1 + \Omega\tau_0 \frac{\partial}{\partial t})\frac{\partial}{\partial t}(\nabla^2 q) + ((1 + \tau_0 \frac{\partial}{\partial t})\frac{\partial}{\partial t} - \nabla^2)T + a_9(1 + \gamma \frac{\partial}{\partial t})\frac{\partial C}{\partial t} = 0, \tag{12}$$

$$a_{10}\nabla^4 q + a_{11}(1 + \tau_1 \frac{\partial}{\partial t})\nabla^2 T + (a_{12}(1 + \Omega\tau^0 \frac{\partial}{\partial t})\frac{\partial}{\partial t} - a_{13}(1 + \tau^1 \frac{\partial}{\partial t})\nabla^2)C = 0, \tag{13}$$

$$(\frac{\partial^2}{\partial t^2} - a_2\nabla^2)\psi = a_3\phi_2, \tag{14}$$

$$(a_5\nabla^2 - a_7 - \frac{\partial^2}{\partial t^2})\phi_2 = a_6\nabla^2\psi, \tag{15}$$

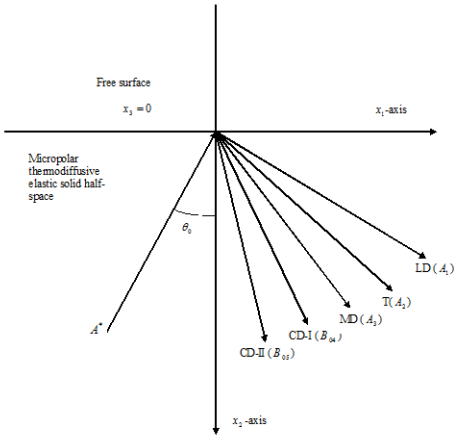

Figure (a) Geometry of the Problem

where

$$a_1 = \frac{\lambda+\mu}{\rho c_1^2},\ a_2 = \frac{\mu+K}{\rho c_1^2},\ a_3 = \frac{K}{\beta_1 T_0},\ a_4 = \frac{\rho c_1^2}{\beta_1 T_0},\ a_5 = \frac{\gamma}{\rho j c_1^2},\ a_6 = \frac{K\beta_1 T_0}{j\rho^2 c_1^2 \omega_1^2},$$

$$a_7 = \frac{2K}{j\rho\omega_1^2},\ a_8 = \frac{\beta_1^3 T_0^2}{K^*\omega_1^2\rho^2 c_1^2},\ a_9 = \frac{a\beta_1 T_0 c_1^2}{K^*\omega_1\beta_2},\ a_{10} = \frac{D\beta_1\beta_2 T_0}{\rho c_1^4},$$

$$a_{11} = \frac{Da\rho}{\beta_1},\ a_{12} = \frac{\rho c_1^2}{\beta_2\omega_1},\ a_{13} = \frac{Db\rho}{\beta_2},\quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.$$

4. REFLECTION AT THE FREE SURFACE

We consider a transverse wave (coupled transverse (CD-I) wave and microrotational (CD-II) wave) propagating through the micropolar generalized thermodiffusion elastic solid half space (x_3 >0) and making an angle of incidence θ_0 with the x_3-axis, at the free surface ($x_3 = 0$). Corresponding to each incident wave, we get waves as reflected LD-Wave, T-wave, MD-Wave, CD-I wave and CD-II wave as shown in Figure (a).

Solutions of the equations (11)-(15) are sought in the form of harmonic travelling wave

$$(q, T, C, \phi_2, \psi) = (\check{q}_0, \check{T}_0, \check{C}_0, \check{\phi}_{2o}, \check{\psi}_0)e^{tk(x_1 \sin\theta - x_3\cos\theta + \upsilon t)}, \tag{16}$$

where υ is the phase velocity, k is the wave number and $(\sin\theta, \cos\theta)$ denotes the projection of the normal onto x_1x_3-plane. Making use of equation (16) in equations (11)-(15), we obtain two equations, one is cubic and second is quadratic in υ^2 and are as follows

$$\upsilon^6 - R\upsilon^4 + Q\upsilon^2 - S = 0 \tag{17}$$

$$\upsilon^4 - A\upsilon^2 + B = 0 \tag{18}$$

where $\upsilon = \frac{\omega}{k}$ is the velocity of the coupled waves; $\upsilon_1, \upsilon_2, \upsilon_3$ are the velocities of the coupled waves namely Longitudinal displacement (LD) wave, Transverse Wave (T), MD-wave and υ_4, υ_5 are the velocities of coupled transverse and microrotational (CD-I and CD-II) waves, respectively given by equation (18), where

$$R = \frac{(a_1 + a_2)a_{12}\tau_g\tau_\theta + \iota\,\omega a_{13}\tau_d\tau_\theta + \iota\,\omega a_9 a_{11}\tau_f\tau_c + \iota\,\omega a_4 a_8 a_{12}\tau_g\tau_f\tau_e + a_{12}\tau_g}{a_{12}\tau_g\tau_\theta}$$

$$Q = \frac{(a_1 + a_2)(a_{12}\tau_g + \iota\,\omega a_{13}\tau_d\tau_\theta + \iota\,\omega a_9 a_{11}\tau_f\tau_c) - \iota\,\omega a_{13}\tau_d(1 + \iota\,\omega a_4 a_8\tau_f\tau_e) + Q_1}{a_{12}\tau_g\tau_\theta}$$

$$S = \frac{(a_1 + a_2)\iota\,\omega a_{13}\tau_d - \iota\,\omega a_4 a_{10}\tau_d}{a_{12}\tau_\theta\tau_g}, \qquad A = \frac{(a_2 + a_5)\omega^2 + a_3 a_6 - a_2 a_7}{\omega^2 - a_7},$$

$$B = \frac{a_2 a_5\omega^2}{\omega^2 - a_7}, \quad Q_1 = \iota\,\omega a_4 a_9 a_{10}\tau_f + \iota\,\omega a_4\tau_d(\iota\omega a_8 a_{11}\tau_f\tau_e - a_{10}\tau_\theta), \quad \tau_\theta = \tau_0 - \frac{\iota}{\omega},$$

$$\tau_c = \gamma_1 - \frac{\iota}{\omega}, \quad \tau_e = \Omega\tau_0 - \frac{\iota}{\omega}, \quad \tau_f = \tau_1 - \frac{\iota}{\omega}, \quad \tau_d = \tau^1 - \frac{\iota}{\omega}, \quad \tau_g = \Omega\tau^0 - \frac{\iota}{\omega},$$

where ι (iota) represents the imaginary unit.

5. BOUNDARY CONDITIONS

The boundary conditions in this case are

$$t_{33} = 0, \qquad t_{31} = 0, \qquad m_{32} = 0, \qquad \frac{\partial T}{\partial x_3} = 0, \qquad \frac{\partial C}{\partial x_3} = 0. \tag{19}$$

In view of above equation, we assume the values of q, T, C, ϕ_2 and ψ satisfying the boundary conditions (19) as

$$\{q, T, C\} = \sum(1, d_i, f_i)(A_{0i}e^{\iota k_i(x_1\sin\theta_0 - x_3\cos\theta_0) + \iota\omega t} + A_i e^{\iota k_i(x_1\sin\theta_i + x_3\cos\theta_i) + \iota\omega t}) \tag{20}$$

$$\{\psi, \phi_2\} = \sum(1, e_n)(B_{0j}e^{\iota k_j(x_1\sin\theta_0 - x_3\cos\theta_0) + \iota\omega t} + B_j e^{\iota k_j(x_1\sin\theta_j + x_3\cos\theta_j) + \iota\omega t}) \tag{21}$$

where

$$d_i = \frac{(a_4 a_{10} - a_{13})\tau_d k_i^4 + (\omega^2 a_{12}\tau_d - \iota\,\omega a_{12}\tau_g)k_i^2 + \iota\,\omega^2 a_{12}\tau_g}{\iota\,\omega a_4(a_{11} - a_{13})\tau_d\tau_f k_i^2 + \iota\,\omega a_4 a_{12}\tau_f\tau_g},$$

$$f_i = \frac{(a_4 a_{10}\tau_d - (a_1 + a_2)a_{11}\tau_f)k_i^4 + \omega^2 a_{11}\tau_f k_i^2}{\iota\,\omega a_4(a_{11} - a_{13})\tau_d\tau_f k_i^2 + \iota\,\omega a_4 a_{12}\tau_f\tau_g},$$

$$e_n = \frac{a_2 k_j - \omega^2}{a_3}, \qquad (i = 1, 2, 3, \quad n = 1, 2\& \quad j = 4, 5)$$

where A_{0i} are the amplitude of the incident LD-wave, T-wave, MD-wave and B_{oj} are the amplitude of the incident(CD-I, CD-II) waves respectively. A_i are the amplitude of the reflected LD-wave, T-wave, MD-wave and B_j are the amplitude of the reflected (CD-I, CD-II) waves respectively.
Snell's law is given as

$$\frac{\sin\theta_0}{\upsilon_0} = \frac{\sin\theta_1}{\upsilon_1} = \frac{\sin\theta_2}{\upsilon_2} = \frac{\sin\theta_3}{\upsilon_3} = \frac{\sin\theta_4}{\upsilon_4} = \frac{\sin\theta_5}{\upsilon_5} \tag{22}$$

where

$$k_1\upsilon_1 = k_2\upsilon_2 = k_3\upsilon_3 = k_4\upsilon_4 = k_5\upsilon_5 = \omega \quad at \quad x_3 = 0 \tag{23}$$

$$\upsilon_0 = \begin{cases} \upsilon_4, & \text{for incident} \quad \text{CD} - \text{I wave} \\ \upsilon_5, & \text{for incident} \quad \text{CD} - \text{II wave} \end{cases}$$

Making use of the potential given by equations (20)-(21) in boundary conditions (19) and using equation (22), we get a system of homogeneous equations which can be written as

$$\sum a_{ij}Z_j = Y_i, \quad (j = 1, 2..., 5), \tag{24}$$

where

$$a_{1i} = (a_1 + h_2)k_i^2(1 - (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0) + h_1k_i^2(\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0 + a_4\iota\omega(\tau_f d_i + \tau_d f_i),$$

$$a_{1j} = (a_1 - h_1 + h_2)k_j^2(\frac{\upsilon_j}{\upsilon_0})^2[(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0,$$

$$a_{2i} = (2h_3 + h_2)k_i^2(\frac{\upsilon_i}{\upsilon_0})^2[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0,$$

$$a_{2j} = -[(h_2 + h_3)(1 - (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0) - h_3(\frac{\upsilon_j}{\upsilon_0})^2 \sin^2\theta_0]k_j^2,$$

$$a_{3i} = \frac{\iota f_i k_i \upsilon_i}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}} \quad a_{3j} = 0 \quad a_{4i} = \frac{\iota d_i k_i \upsilon_i}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}},$$

$$a_{4j} = 0, \quad a_{5i} = 0, \; a_{5j} = e_n\iota k_j\frac{\upsilon_j}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \; h_1 = \frac{\lambda}{\rho c_1^2}, \; h_2 = \frac{K}{\rho c_1^2},$$

$$h_3 = \frac{\mu}{\rho c_1^2}, \; Z_i = \frac{A_i}{A^*}, \; Z_j = \frac{A_j}{A^*}, \qquad (i = 1, 2, 3, \; j = 4, 5, \; \& n = 1, 2)$$

where Z_i and Z_j are complex amplitude ratio's of reflected LD wave, T wave, MD wave, CD-I wave and CD-II wave respectively.
Considering the phase of the reflected waves can easily be written using equations (22)-(23)

$$\frac{\cos\theta_i}{\upsilon_i} = [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad \frac{\cos\theta_j}{\upsilon_j} = [(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}} \quad (i = 1, 2, 3 \& j = 4, 5).$$

Following Schoenberg [30], if we write

$$\frac{\cos\theta_i}{\upsilon_i} = \frac{\cos\theta_i'}{\upsilon_i'} + \iota\frac{c_i}{2\pi\upsilon_0} \qquad (i = 1, 2, 3, 4, 5) \text{ then}$$

$$\frac{\cos\theta_i'}{\upsilon_i'} = \frac{1}{\upsilon_0}R_e\{[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}\}, \quad c_i = 2\pi I_m\{[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}\}$$

where υ_i', the real phase speed and θ_i', the angle of reflection are given by

$$\frac{\upsilon_i'}{\upsilon_0} = \frac{\sin\theta_i'}{\sin\theta_0}[\sin^2\theta_0 + [R_e\{[(\upsilon_0/\upsilon_5)^2 - \sin^2\theta_0]^{\frac{1}{2}}\}]^2]^{-\frac{1}{2}}$$

and c_i, the attenuation in a depth is equal to the wavelength of incident wave i.e. $(2\pi\upsilon_0)/\omega$

For Incident CD-I wave: $A^* = B_{04}$

$$Y_1 = a_{14}, \quad Y_2 = -a_{24}, \quad Y_3 = a_{34}, \quad Y_4 = a_{44}, \quad Y_5 = a_{54}$$

For Incident CD-II wave: $A^* = B_{05}$

$$Y_1 = a_{15}, \quad Y_2 = -a_{25}, \quad Y_3 = a_{35}, \quad Y_4 = a_{45}, \quad Y_5 = a_{55}$$

6. PARTICULAR CASES

(6.1) **Neglecting diffusion effect** ($\beta_2 = b = a = 0$) : we obtain the corresponding expression for micropolar thermoelastic with two relaxation times half space as

$$\sum a_{ij}Z_j = Y_i, \quad (j = 1, 2..., 4), \tag{25}$$

where

$$a_{1i} = (a_1 + h_2)k_i^2(1 - (\frac{\upsilon_i}{\upsilon_0})^2\sin^2\theta_0) + h_1k_i^2(\frac{\upsilon_i}{\upsilon_0})^2\sin^2\theta_0 + a_4\iota\omega\tau_f d_i,$$

$$a_{1j} = (a_1 - h_1 + h_2)k_j^2(\frac{\upsilon_j}{\upsilon_0})^2[(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}}\sin\theta_0,$$

$$a_{2i} = (2h_3 + h_2)k_i^2(\frac{\upsilon_i}{\upsilon_0})^2[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}\sin\theta_0,$$

$$a_{2j} = -[(h_2 + h_3)(1 - (\frac{\upsilon_j}{\upsilon_0})^2\sin^2\theta_0) - h_3(\frac{\upsilon_j}{\upsilon_0})^2\sin^2\theta_0]k_j^2,$$

$$a_{4i} = \frac{\iota d_i k_i \upsilon_i}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}a_{4j} = 0, a_{5i} = 0,$$

$$a_{5j} = e_n\iota k_j\frac{\upsilon_j}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}}, (i = 1, 2 \quad j = 4, 5)$$

For Incident CD-I wave: $A^* = B_{04}$

$$Y_1 = -a_{14}, \quad Y_2 = -a_{24}, \quad Y_4 = a_{44}, \quad Y_5 = a_{54}$$

For Incident CD-II wave: $A^* = B_{05}$

$$Y_1 = -a_{15}, \quad Y_2 = a_{25}, \quad Y_4 = a_{45}, \quad Y_5 = a_{55}$$

and the values of υ_1, υ_2 are obtained from equation

$$(\upsilon^4 + M\upsilon^2 + N)(\widetilde{q}, \widetilde{T}) = 0 \tag{26}$$

$$M = \frac{\omega^2 + 1}{a_4 a_8 + \omega^2}, \quad N = \frac{\omega^2}{a_4 a_8 + \omega^2}$$

The above results tally with those obtain by Kumar and Singh [31]

(6.2)**Neglecting Micropolarity effect** ($K = 0, j = 0$): we obtain the corresponding expression for thermodiffusion elastic solid half space as

$$\sum a_{ij} Z_j = Y_i, \quad (j = 1, 2..., 4), \tag{27}$$

where

$$a_{1i} = a_1 k_i^2 (1 - (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0) + h_1 k_i^2 (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0 + a_4(d_i\tau_f - f_i\tau_d),$$

$$a_{14} = (a_1 - h_1) k_4^2 (\frac{\upsilon_4}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_4})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0,$$

$$a_{2i} = 2h_3 k_i^2 (\frac{\upsilon_i}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0, \; a_{24} = -h_3[1 - 2(\frac{\upsilon_4}{\upsilon_0})^2 \sin^2\theta_0] k_4^2,$$

$$a_{3i} = \iota f_i k_i \frac{\upsilon_i}{\upsilon_0} [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad a_{34} = 0, \quad a_{4i} = \iota d_i k_i \frac{\upsilon_i}{\upsilon_0} [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}},$$

$$a_{44} = 0, \quad Z_i = \frac{A_i}{A^*}, \quad Z_4 = \frac{A_4}{A^*}, \quad (i = 1, 2, 3)$$

For Incident CD-I wave: $A^* = B_{04}$

$$Y_1 = a_{14}, \quad Y_2 = -a_{24}, \quad Y_3 = a_{34}, \quad Y_4 = a_{44}$$

and the values of υ_4 are obtained from equation

$$\upsilon^2 - a_2 = 0 \tag{28}$$

with changed values of

$$a_2 = \frac{\mu}{\rho c_1^2}$$

The above results are similar as obtained by Singh [15]

7. SPECIAL CASE

Neglecting Relaxation times i.e. $\tau_0 = \tau^0 = \tau_1 = \tau^1$=0 in equations (11)-(15), we obtain the corresponding expressions for the coupled theory of thermoelasticity.

8. NUMERICAL RESULT AND DISCUSSION

In order to illustrate theoretical results obtained in proceeding section, we now present some numerical results. The material parameter chosen for this purpose are Micropolar parameter Eringen [32]

$$\lambda = 9.4 \times 10^{10} N\ m^{-2}, \quad \mu = 4.0 \times 10^{10} N\ m^{-2}, \quad K = 1.0 \times 10^{10} N\ m^{-2}$$

$$\gamma = 0.779 \times 10^{-9} N, \quad \rho = 1.74 \times 10^{3} Kg\ m^{-3},$$

Thermodiffusion parameter Thomas [33]

$$C_E = 1.0 J\ kg^{-1}\ deg^{-1}, \quad K^* = 1.7 \times 10^2 J\ m^{-1}\ sec^{-1}\ deg^{-1}, \quad T_0 = 298\ K,$$

$$\alpha_t = 1.78 \times 10^{-5} K^{-1}, \quad \alpha_c = 1.98 \times 10^{-4} m^3\ kg^{-1}, \quad a = 1.2 \times 10^4 m^2\ s^{-2}\ K^{-1},$$

$$b = 0.9 \times 10^6 m^5\ kg^{-1}\ s^{-2}, \quad j = 0.2 \times 10^{-19} m^2, \quad D = 0.85 \times 10^{-8} kg\ s\ m^{-3}.$$

The solid line corresponds for LMTD (micropolar thermodiffusion), dashed line indicates LMT (micropolar thermoelasticity) and small dashed line represents LTD (Thermodiffusion elastic) for L-S model and the solid line with centre symbol ”triangle” corresponds for GMTD (micropolar thermodiffusion), dashed line with centre symbol ”circle” indicates GMT (micropolar thermoelasticity) and small dashed line with centre symbol ”diamond” represents GTD (Thermodiffusion elastic) for G-L model respectively against the angle of incidence $0^0 \leq \theta \leq 90^0$

8.1. CD-I WAVE

Figure 1 depicts the variations of $|Z_1|$ with θ. It is noticed that values of $|Z_1|$ for LMTD and GMTD increases till θ=18, then decreases towards origin, magnitude of LMTD is greater than GMTD. Also values of $|Z_1|$ for LMT and GMT increases in first half of interval and vice-versa in rest of interval, magnitude of values for LMT are greater than GMT. Also values of $|Z_1|$ for GTD increases till θ=45 and then decreases in remaining range whereas LTD shows steady state about 0.5 which is accounted as absence of micropolarity.

The variations of $|Z_2|$ with θ are depicted in Figure 2. It is noticed that values of $|Z_2|$ for LMTD, LMT, GMTD and GMT increase in first half of interval and vice-versa trends are noticed in remaining range, magnitude of value of $|Z_2|$ for G-L model is greater than L-S model, while values of $|Z_2|$ for LTD increases till θ=60 and decreases in rest of interval, which clearly indicates the impact of micropolarity in L-S theory.

Figure 3 depicts the variations of $|Z_3|$ with θ. It is noticed that values of $|Z_3|$ for L-S theory i.e. LMTD and LTD increases with greater magnitude as compared to G-L

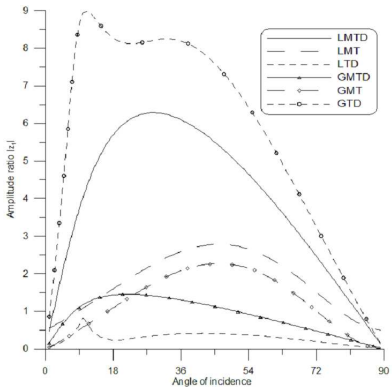

Fig. 1.

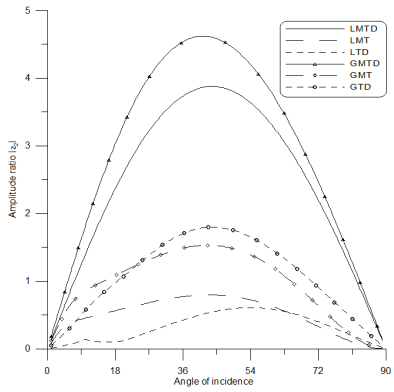

Fig. 2.

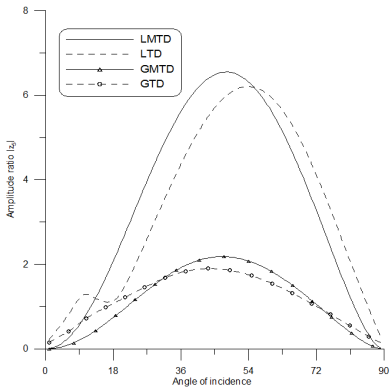

Fig. 3.

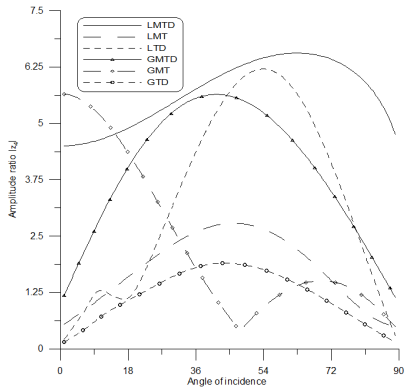

Fig. 4.

theory, when $0 \leq \theta \leq 45$ and after that values of $|Z_3|$ for both theories of thermoelasticity decreases.
The variations of $|Z_4|$ with θ are noticed in Figure 4. It is noticed that values of $|Z_4|$ for GMTD increase in first half and vice-versa in second half, while value of $|Z_4|$ for LMTD increases, when $0 \leq \theta \leq 70$ and then decreases which is accounted as absence of second relaxation times in L-S model, whereas values of $|Z_4|$ for LTD decreases in range $9 \leq \theta \leq 18$, $55 \leq \theta \leq 90$ and increases in remaining range. Also values of $|Z_4|$ for GTD increase as θ increases to 45 and after that opposite behavior is noticed. Whereas LMT and GMT shows opposite behavior in entire range, which reveals the impact of diffusion parameters.

It is noticed from Figure 5, which is a plot of $|Z_5|$ with θ that the trends of variations for G-L theory of thermoelasticity i.e. GMTD and GMT are similar as noticed for L-S theory i.e. LMTD and LMT, magnitude of values for G-L theory is greater as compared to L-S theory, which clearly shows the impact of relaxation times.

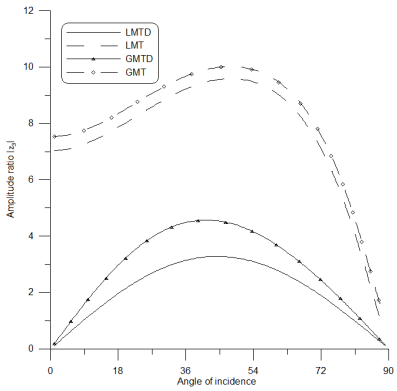

Fig. 5.

8.2. CD-II WAVE

The variations of $|Z_1|$ with θ are presented in Figure 6. The values of $|Z_1|$ for LMTD and GMTD increases for $0 \leq \theta \leq 10$ and then decreases towards origin as θ increases, whereas values of $|Z_1|$ for LMT and GMT increases in first half of interval, which is accounted as absence of diffusion effect and vice-versa is noticed in remaining range. Figure 7 depict the trend of variations of $|Z_2|$. It is noticed that pattern for LMT and GMT are similar in nature in entire range i.e. their values increase in range $0 \leq \theta \leq 10$, $30 \leq \theta \leq 50$ and decrease in remaining range, magnitude for GMT being greater. Also trends of $|Z_2|$ for LMTD and GMTD are similar to those observed for $|Z_1|$, magnitude being different.

Figure 8 depicts the variations of $|Z_3|$ with θ. It is noticed that values of $|Z_3|$ for GMTD increases as θ increases to 18 and decreases in remaining range, whereas value of $|Z_3|$ for LMTD increases till θ=45, which is due to absence of relaxation times and decreases in remaining range.

The variations of $|Z_4|$ with θ are depicted in Figure 9. It is noticed that values of $|Z_4|$ for GMTD and LMTD increases monotonically for $0 \leq \theta \leq 60$ and reversed trends are noticed after that. Also similar trends are noticed for both LMT and GMT, magnitude of values for G-L theory are greater as compared to L-S theory.

The trend of variations for $|Z_5|$ with θ are persented in Figure 10. It is noticed that trends of $|Z_5|$ for LMTD and GMTD have similar behavior in entire range except in interval $25^0 \leq \theta \leq 40^0$ where reverse trends are noticed , whereas as similar trends of $|Z_5|$ are noticed for LMT and GMT in entire range which is mainly due to absence diffusion effect, magnitude of values for LMT being greater or compared to GMT.

9. CONCLUSIONS

The most significant conclusion which emerges out from the above numerical discussion is that reflection is influenced by the presence of micropolarity and thermod-

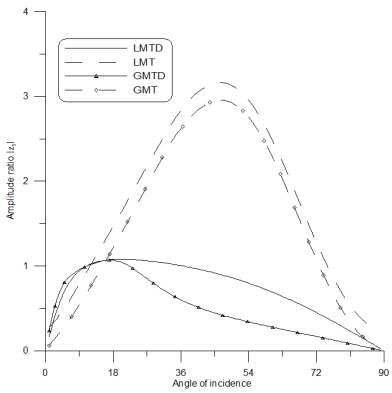

Fig. 6.

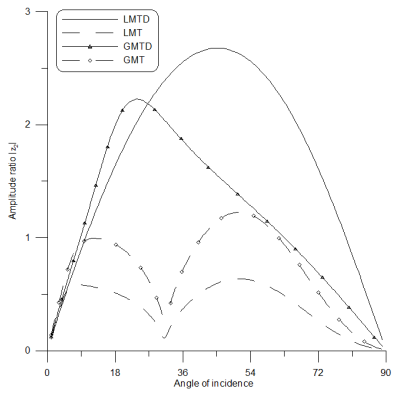

Fig. 7.

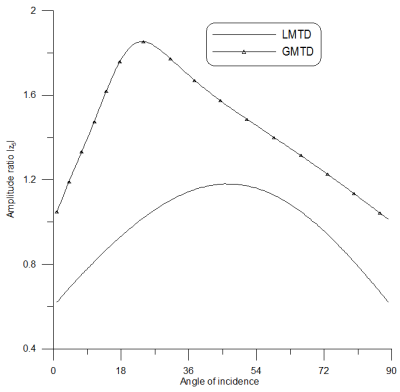

Fig. 8.

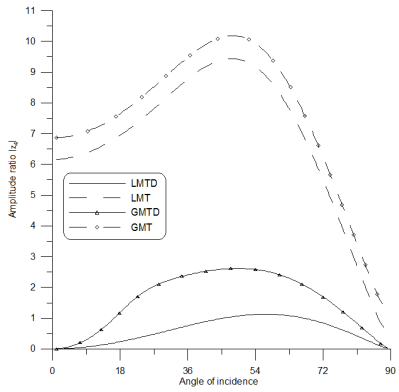

Fig. 9.

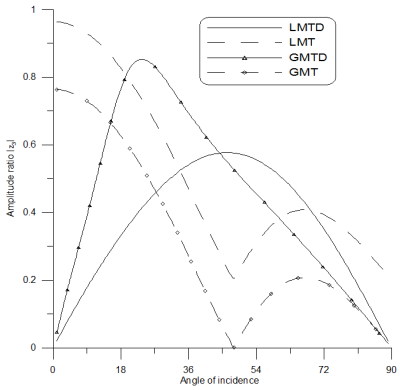

Fig. 10.

iffusion effects. It is observed from the figures (1)-(10), that the impact of micropolarity on amplitude ratios is more as compared to thermodiffusion i.e. trends for thermodiffusion is similar to that noticed for micropolar thermodiffusive in most of the figures with significant difference in their magnitude. The problem though is theoretical but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering and seismologist working in the field of mining tremors and drilling into the crust of the earth. Also this theory has industrial utilities in improving the conditions for extraction of oil due to study of phenomenon of diffusion.

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