ROMAI J., v.10, no.1(2014), 137–145

THE SYNCHRONIZATION OF TWO CHAOTIC MODELS OF CHEMICAL REACTIONS

Servilia Oancea[1], Andrei-Victor Oancea[2], Ioan Grosu[3]
[1] *U.S.A.M.V., Iaşi, Romania*
[2] *Erasmus Mundus Advanced Spectroscopy in Chemistry Master, Bologna University, Bologna, Italy*
[3] *"Al.I.Cuza" University, Iaşi, Romania*
liaoancea@yahoo.com, victorashoancea@yahoo.com, grosu45@yahoo.co.uk

Abstract The main goal of this paper is to study the synchronization of two identical chemical chaotic systems proposed by Peng and coworkers, which is based on six rections and three intermediary species, using an adaptive feedback method. The transient time until synchronization depends on initial conditions of two systems, the strength and the number of the controllers. To achieve the synchronization is absolutely necessary to use two controllers, for example in the second and in the third equation from the slave system; if the time of synchronization might be longer than the previous case we can use only one controller, applied in the second equation of the tridimensional system.

Keywords: domains with perturbed boundaries; Laplace operator; boundary eigenvalue problems; perturbation and pseudoperturbediteration methods, regularization.
2010 MSC: 47A55, 35B20.

1. INTRODUCTION

Many physical, chemical, biological processes exhibit nonlinear dynamic characteristics, in the sense of the dependence between the magnitude of the driving forces and the intensity of the resulting flows. Chemical systems can exhibit specific behaviour and this fact is very important for chemical processes and for biological structures. Among such chemical reactions, the classical Belousov - Zhabotinsky (BZ) reaction is one of the best known systems. The BZ reaction is a family of oscillating chemical reactions; in these reactions bromate ions are reduced in an acidic medium by an organic compound (usually malonic acid) with or without a catalyst (usually cerous and/or ferrous ions). Due to the fact that these reactions are autocatalytic, the rate equations are fundamentally nonlinear. This nonlinearity can lead to the spontaneous generation of order and chaos. Schmitz, Graziani, and Hudson were the first to report observations of chaos in a chemical reaction. They conducted an experiment on the BZ reaction in a continuous flow stirred tank reactor (CSTR), where the flow rate of the feed chemicals was maintained constant and the reaction behavior was monitored with bromide ion specific and platinum wire electrodes. Ilya Prigogine argued that, far from thermodynamic equilibrium, qualitatively new behav-

iors appear as the system enters new dynamical regimes. From this point of view the deliberate control of these phenomena have a great practical impact despite the fact that it is very dificult; this is the reason the theoretical models are useful in these situations. In addition, the control using these models can give the informations about the selfcontrol inside the biological structures where the behaviour of the dynamic systems is realized by a feedback mechanism. Over the last decade, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators especially from technical reasons. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic. Many examples of synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies [1-6]. The main goal of this paper is to study the synchronization of two chemical chaotic systems based on the adaptive feedback method of control. One of these chemical models was proposed by Peng et al. [7] and it is based on six reactions and three intermediary species. This system consists in the following elementary steps:

1. $P \to A$
2. $P + C \to A + C$
3. $A \to B$
4. $A + 2B \to 3B$
5. $B \to C$
6. $C \to D$

The rate equations for these autocatalytic reactions, having three intermediary species A, B and C are:

$$\begin{aligned} \frac{dA}{dt} &= k_1 PC - k_3 A - k_4 AB^2 \\ \frac{dB}{dt} &= k_3 A + k_4 AB^2 - k_5 B \\ \frac{dC}{dt} &= k_4 B - k_3 C \end{aligned}$$

This system was simplified by Andrievskii and Fradkov [8] and the time evolution of the intermediary species x_1, x_2 and x_3 is given by the nonlinear system of equations into dimensionless form:

$$\begin{aligned} \frac{dx_1}{dt} &= \mu k + x_3) - x_1 x_2^2 - x_1 \\ \frac{dx_2}{dt} &= \frac{1}{\sigma}(x_1 x_2^2 + x_1 - x_2) \\ \frac{dx_3}{dt} &= \frac{1}{}\delta(x_2 - x_3). \end{aligned} \tag{1}$$

This system has a chaotic behavior, for the following constants: $\sigma = 0.015$, $\delta = 1$, $\mu = 0.301$ and $\kappa = 2.5$.

2. CHAOTIC DYNAMICS OF CHEMICAL SYSTEM

The strange attractor for this system is given in the Figure 1. The dynamics of this

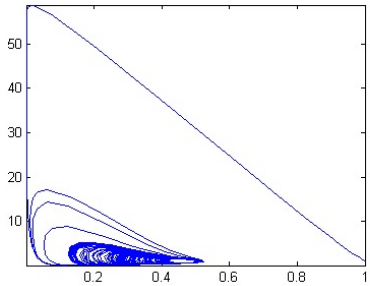

Fig. 1.: The 2D attractor with initial conditions $x_1(0) = x_2(0) = x_3(0) = 1$

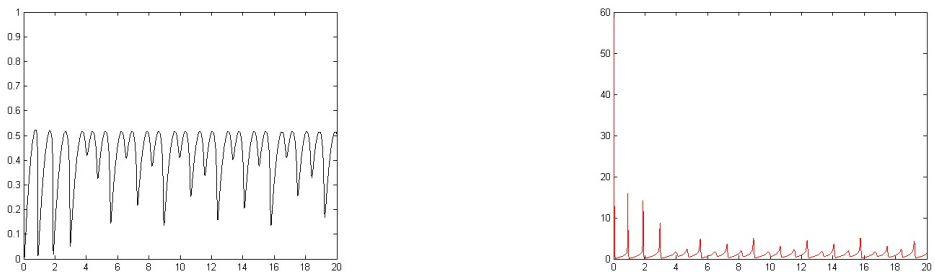

Fig. 2.: a-$x_1(t)$; b- $x_2(t)$ for initial conditions $x_1(0) = x_2(0) = x_3(0) = 1$

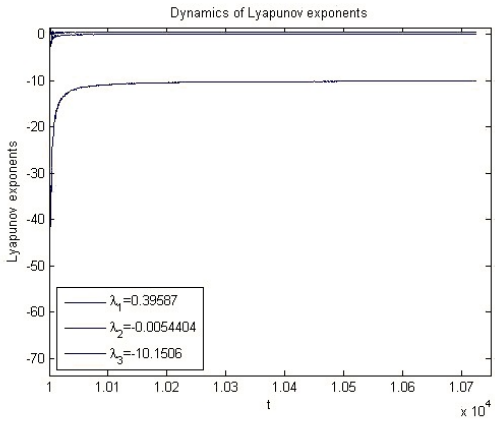

Fig. 3.: The Lyapunov exponents

chaotic chemical system is given in Figure 2. The chaotic behavior is sustained by Lyapunov exponents from Figure 3 From Figure 3 we can see that one of Lyapunov exponents is positive; that means this system is chaotic for given constants.

3. THE SYNCHRONIZATION OF TWO CHAOTIC SYSTEMS

To synchronize two identical chemical systems we followed the method proposed by D. Huang [9], Hu and Xu [10], Guo et al. [11], Oancea et al. [12] based on Lyapunov-Lasalle theory.

Let be a chaotic non-autonomous :

$$\dot{x} = f(t, x) \quad \text{where} \;\; x = (x_1, x_2, ...)^T \in \mathbb{R}^n \tag{2}$$

is the state vector of the system and $f = (f_1, f_2, ...)^T \in \mathbb{R}^n$ is the non-linear vector field of the system, which is considered as a driving system. For any $x = (x_1, x_2, ...)^T \in \mathbb{R}^n$ and $y = (y_1, y_2, ...)^T \in \mathbb{R}^n$ there exists a positive constant ℓ such that:

$$|f(x, t) - f(y, t)| \leq \ell \; max|x_i - y_j| \quad i, j = 1, 2, ..., n.$$

The slave system will be:

$$\dot{y} = f(y, t) + z(z_1, z_2, ...) \tag{3}$$

where $z(z_1, z_2, ...)$ is the controller. If the error vector is $e = y - x$, the objective of synchronization is to make

$$\lim |e(t)| \to 0 \;\; for \;\; t \to +\infty$$

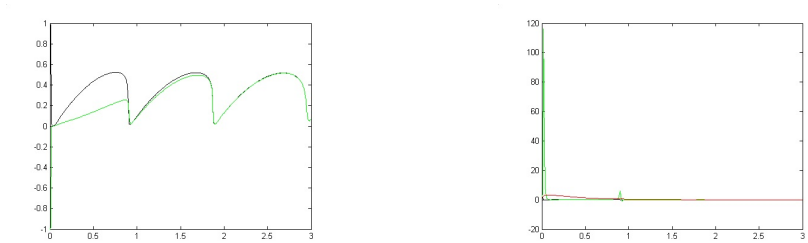

Fig. 4.: a-$x_1(t)$- black $y_1(t)$- green; b- Synchronization errors between master and slave systems $[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1; y_1(0) = -1; y_2(0) = -1, y_3(0) = -1; z_1(0) = 1; z_2(0) = 1; z_3(0) = 1]$

The controller is of the form:

$$z_i = \varepsilon_i(x_i - y_i) \tag{4}$$

and

$$\dot{\varepsilon}_i = -\gamma_i e_i^2, \;\; i = 1, 2, ..., n \;\text{ and } \gamma_i, \; i = 1, 2, ..., n \tag{5}$$

are arbitrary positive constants.

4. THE SYNCHRONIZATION OF TWO CHAOTIC CHEMICAL SYSTEMS

According this method of synchronization, the slave system for the system (1) will be:

$$\begin{aligned} \frac{dy_1}{dt} &= 0.30\ell(k + y_3) - y_1 y_2^2 - x_1 + z_1(y_1 - x_1) \\ \frac{dy_2}{dt} &= \frac{\ell}{0.015}(y_1 y_2^2 + y_1 - y_2 + z_2(y_2 - x_2)) \\ \frac{dy_3}{dt} &= (y_2 - y_3) + z_3(y_3 - x_3). \end{aligned} \tag{6}$$

and the control strength:

$$\begin{aligned} \dot{z}_1 &= -(y_1 - x_1)^2 \\ \dot{z}_2 &= -(y_2 - x_2)^2 \\ \dot{z}_3 &= -(y_3 - x_3)^2. \end{aligned} \tag{7}$$

Figures 4-6 demonstrate the synchronization of the two chemical systems

From practical point of view, the synchronization using a single controller is of interest. We obtained such synchronization for Lorenz system adding one controller in any equation of the three-dimensional system [6]. D. Huang [9], by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that it can use a single controller to achieve identical synchronization of a three-dimensional system. For Lorenz system this is possible only by adding the controller in the second equation.

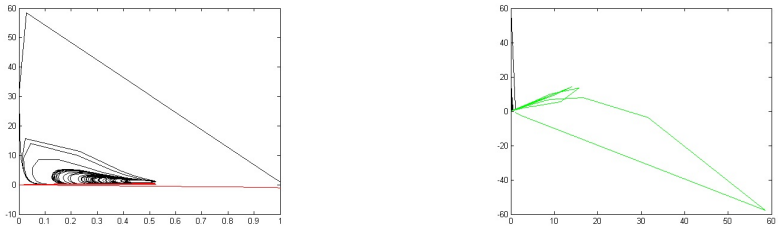

Fig. 5.: Phase portrait a) - (x_1, x_2-black) and (x_1, y_1-red); b) - (x_1, x_2)-black) and (x_2, y_2-green) for two systems $[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1; y_1(0) = 1.1; y_2(0) = 1.1,\ y_3(0) = 1.1; z_1(0) = 1; z_2(0) = 1; z_3(0) = 1]$

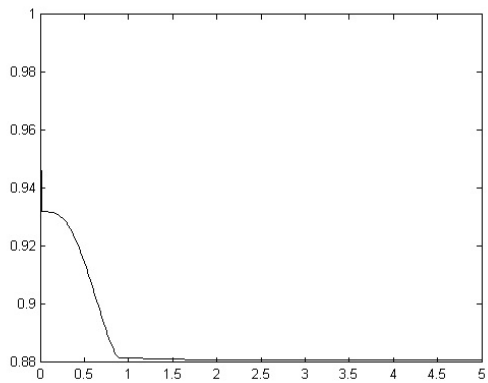

Fig. 6.: The control strength $z_1(t)[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1; y_1(0) = 1.1; y_2(0) = 1.1, y_3(0) = 1.1; z_1(0) = 1; z_2(0) = 1; z_3(0) = 1]$

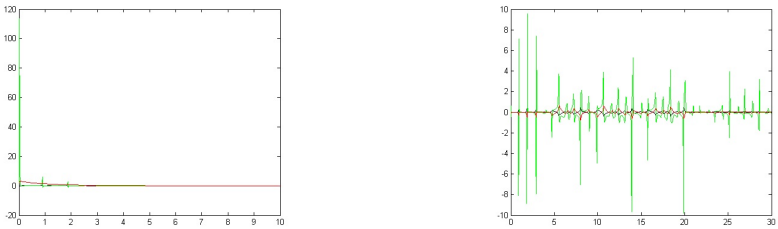

Fig. 7.: a - Synchronization errors between master and slave for chemical systems with one controller z_2; b - Synchronization errors between master and slave for chemical systems with one controller z_3; $[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1; y_1(0) = -1; y_2(0) = -1,\ y_3(0) = -1; z_2(0) = 1, z_3(0) = 1]$

For the systems (1), (6) an (7) we achieved the synchronization if one controller is applied only in the second equation (Figure 7a). From Figures 4b and 7a we can see that the synchronization is obtained two times later if one controller was applied in second equation of the slave system. Despite the fact that we used the very closed initial conditions for master and slave, the synchronization is not achieved when the one controller is applied in the first or the third equation of the slave system (Figure 7b). Using two controllers, in the second and in the third equations, the synchronization is obtained very fast (2 unities of time), as in the case when all the controllers are used in the slave system (Figure 8).

5. CONCLUSIONS

In this work we analyzed the dynamics of Peng and coworkers system which is based on six reactions and three intermediary species. We performed the synchronization of two systems using an adaptive feedback method. The transient time until synchronization depends on initial conditions of two systems, the strength and the number of the controllers. From practical point of view, the synchronization using a single controller is of interest. To meet this goal we tried to achieve the synchronization using a single controller in any equation of the system but this isn't possible; only one controller is applied in the second equation of the 3-dimensional system the synchronization is achieved. In this case the time until synchronization is longer than the all the controllers are applied. Then is absolutely necessary to use even two controllers (in the second and in the third equation from the slave system); if the time of synchronization might be longer than the previous case we can use only one controller, applied in the second equation of the 3-dimensional system.

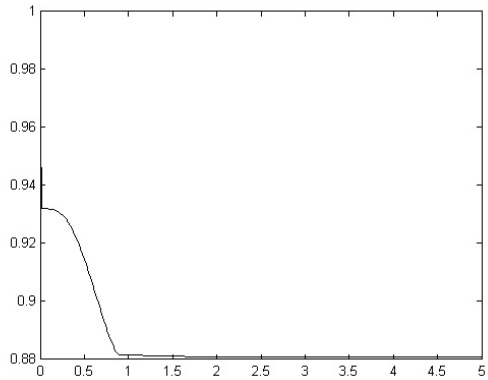

Fig. 8.: Synchronization errors between master and slave for chemical systems with two controllers $[x_1(0) = 1, x_2(0) = 1, x_3(0) = 1; y_1(0) = -1; y_2(0) = -1 y_3(0) = -1; z_2(0) = 1;\ z_3(0) = 1]$

References

[1] Chen G, Dong X., *From chaos to order: methodologies, perspectives and applications*, World Scientific: Singapore, 1998.

[2] Grosu I., *Robust Synchronization*, Phys. Rev. 56, 1997, 3709-3712.

[3] Lerescu A.I., Constandache N., Oancea S., Grosu I., *Collection of master-slave synchronized chaotic systems*, Chaos Soliton and Fractals, 22(3), 2004, 599-604.

[4] Lerescu A.I., Oancea S., Grosu I., *Collection of Mutually Synchronized Chaotic Systems*, Physics Letters A, 352, 2006, 222-228.

[5] Grosu I., Padmanaban E., Roy P. K., Dana S. K., *Designing Coupling for Synchronization and Amplification of Chaos*, Phys Rev Lett 100, 234102, 2008, 1-4.

[6] Oancea S., Grosu F., Lazar A., Grosu I., *Master - slave synchronization of Lorenz systems using a single controller*, Chaos, Solitons and Fractals, 41, 2009, 2575-2580.

[7] Peng B., Petrov V., Showalter K., *Controlling Chemical Chaos*, J. Phys. Chem. 95, 1991, 4957-4959.

[8] Andrievskii B.R., Fradkov A.L., *Control of Chaos: Methods and Applications II*, Applications, Automation and Remote Control, 65(4), 2004, 505-533.

[9] Huang D., *Simple adaptive-feedback controller for identical chaos synchronization*, Phys. Rev. E, 71, 2005, 037203

[10] Hu M., Xu Z., *Adaptive feedback controller for projective synchronization*, Nonlinear Analysis: Real World Applications, 9, 2008, 1253 - 1260

[11] Guo W., Chen S., Zhou H., *A simple adaptive-feedback controller for chaos Synchronization*, Chaos, Solitons and Fractals 39, 2009, 316-321

[12] Oancea S., Grosu I., Oancea A.V., *Biological control based on the synchronization of Lotka-Volterra systems with four competitive species*, Rom. J. Biophys, 21(1), 2011, 17-26

ROMAI J., v.10, no.1(2014), 145–155

THE THEORETICAL APPROACH OF FACE SEALS PRESSURE FOR HYDRODYNAMIC OPERATING MODEL

Constantin Onescu, Nicolae Popa, Marin Nicolae Popescu,
Alisa Filofteia Oprea
University of Piteşti, Piteşti, Romania
constantin.onescu@upit.ro, npopa49@yahoo.com, ofaly73@yahoo.com

Abstract The power series approach is used to solve the Reynolds equation for hydrodynamic face seal lubrication. The angular misalignment of the stator and rotor is considered. Obtaining another set of partial differential equations, the Fourier series approaches the pressure components. Solving the particulars non-homogenous Euler-Cauchy equations, the analytical expression of hydrodynamic film pressure between the seal rings is computed.

Keywords: face seal, Reynolds equation, power series, Fourier series.
2010 MSC: 97U99.

1. INTRODUCTION

The face seals are used within the mechanical systems where efficient operation is required. This is the reason for that physicists, mathematicians and engineers are interested by the face seals study.

The face seal operation is conditioned by minimum friction losses, that means the full separation of the ring surfaces. By the other way, a great interspace supposes great seal leakage. The pressure distribution within the fluid supposes a separation field model [1]. The simplest model is considered the model with flat, smooth and parallel surfaces, with uniform film and linear pressure distribution. The nature of the film was the subject of many researches, most of authors considering hydrodynamic behaviour [1, 2]. The thin film hydrodynamic approach was introduced by O. Reynolds and this is the fundamental basis for the theoretical studies [4].

To solve Reynolds equation the numerical methods are widely used, as shown in [3], but the analytical solution is important from seal optimisation point of view. In [5] the pressure was approached by Fourier series and was obtained a set of ordinary differential equations, easy to solve. Pressure, load capacity and leakage rate are plotted depending by the tilt parameter. To solve analytically the Reynolds equation in cylindrical coordinates, two approaches are considered: power series and Fourier series. The theory behind the power series method to solve differential equations is rather simple but the algebraic procedures involved could be quite complex, partic-