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MODIFIED HAN ALGORITHM FOR MARITIME CONTAINERS TRANSPORTATION PROBLEM

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Abstract The most important particularity of the maritime container transportation is the fact that the containers have different capacities, so the transportation cost of a unit of cargo is not the same for each container for a particular route. If we consider the cost of transport of a Twenty-foot Equivalent Unit (TEU, for short) between the supplier to the warehouse, the difference consists in that the costs are for TEU and not for each container. The mathematical model of a real world context will provide us an inconsistent transportation problem. In this respect, we reformulate the problem as an inconsistent (incompatible) system of linear inequalities, for which we propose a modified version of Han's iterative algorithm. Numerical experiments and comparisons with the classical Simplex algorithm are presented.

Keywords: inconsistent linear inequalities; least squares solutions; Han-type algorithms; maritime containers transportation problem.

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1. INTRODUCTION

The classical transportation problem involves sources $(S_i)_{i\in\{1,\dots,n\}}$, where supplies $(s_i)_i = 1,\dots,n$ of some goods are available, and destinations $(D_j)_{j\in\{1,\dots,m\}}$, where some demands $(d_j)_{j=1,\dots,m}$ are requested (see for details [7]). The costs of shipping $(c_{ij})_{i\in\{1,\dots,n\},j\in\{1,\dots,m\}}$ for the transportation of one unit from source S_i to destination D_j become the entries of the $C : n \times m$ cost matrix (see Table 1.1).

If we denote by $x_{ij}, i = 1,\dots,n, j = 1,\dots,m$ the number of units transported from source S_i to destination D_j, we get the following mathematical model of the (classical) transportation problem:

Table 1: The classical transportation problem

	D_1	D_2	D_3	$\dots$	D_m	Supply(s)
S_1	c_{11}	c_{12}	c_{13}	$\dots$	c_{1m}	s_1
S_2	c_{21}	c_{22}	c_{23}	$\dots$	c_{2m}	s_2
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
S_n	c_{n1}	c_{n2}	c_{n3}	$\dots$	c_{nm}	s_n
Demand(d)	d_1	d_2	d_3	$\dots$	d_m	

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} && (1)\\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ij} \geq d_j, j = 1,\dots,m && (*)\\
& \sum_{j=1}^{m} x_{ij} = s_i, i = 1,\dots,n && (**)\\
& x_{ij} \geq 0, i = 1,\dots,n, j = 1,\dots m,
\end{aligned}$$

Remark 1. *Some arguments for the relations (*)-(**) are as follows:*
- for (*): at each destination, the demand has to be "at least" satisfied (e.g. the construction of a building will not be started if we do not have at least a minimal amount of materials)
- for (**): all available units must be supplied.

The problem is called **balanced** if the total supply equals the total demand (i.e. $\sum_{i=1}^{n} s_i = \sum_{j=1}^{m} d_j$), and **unbalanced** otherwise. In the balanced case or the unbalanced one with $\sum_{i=1}^{n} s_i \geq \sum_{j=1}^{m} d_j$, the linear program (1) is consistent and well known methods (including Simplex-type algorithms) are available (see [5, 9, 10]). We will consider in this paper the unbalanced case

$$\sum_{i=1}^{n} s_i < \sum_{j=1}^{m} d_j \tag{2}$$

for which the linear program (1) becomes inconsistent (i.e. the set of feasible solutions is empty).

2. MARITIME CONTAINER TRANSPORTATION PROBLEM

2.1. TYPES OF CONTAINERS

The maritime container transportation is a transportation problem subject to the following additional hypothesis (see e.g. [8]):
(**M1**) the unit of cargo has different capacities, so the cost of a unit of transport is different for a particular route. The most common unit used in cargo transportation is TEU (Twenty-foot Equivalent Unit), corresponding to a 20-foot-long (6.1 m) intermodal container. Other types of containers (expressed in TEU) are shown in Table 2.1.
(**M2**) the destinations become in this case warehouses with specific dimensions; hence, the number of containers transported to a given warehouse will be restricted by these dimensions.
Details for (**M2**): we will assume that all the warehouses are rectangular buildings,

Table 2: Types of containers

Abbreviation	Length	Width	Height	Volume	TEU
Type(1)	20ft(6.1m)	8ft	8ft6in(2.59m)	1,360 cu ft ($38.5m^3$)	1
Type(2)	40ft(12.2m)	8ft	8ft6in(2.59m)	2,720 cu ft ($77m^3$)	2
Type(3)	45ft(13.7m)	8ft	8ft6in(2.59m)	3,060 cu ft ($86.6m^3$)	2 or 2.25
Type(4)	48ft(14.6m)	8ft	8ft6in(2.59m)	3,264 cu ft ($92.4m^3$)	2.4
Type(5)	53ft(16.2m)	8ft	8ft6in(2.59m)	3,604 cu ft ($102.1m^3$)	2.65
Type(6) (High cube)	20ft(6.1m)	8ft	9ft6in(2.90 m)	1,520 cu ft ($43m^3$)	1
Type(7) (Half height)	20ft(6.1m)	8ft	4ft3in(1.30 m)	680 cu ft ($19.3m^3$)	1

redhttp://en.wikipedia.org/wiki/Twenty-foot_equivalent_unit

with length (L), width (W) and height(H). Thus, the containers will be stored on superposed rows in each warehouse. In the next section we will present on a model problem, a mathematical model for the problem of containers' storage.

2.2. A MODEL PROBLEM

Since the standard width and height of the containers used in practice are 2.44 and 2.59, respectively (see e.g. [8, 10]), and taking into consideration the handling procedures in the warehouses, we will use for the number of rows ($R(j)$) and columns ($C(j)$) of containers stored in a warehouse the formulas

$$R(j) = \left[\frac{W_j}{3}\right], \quad C(j) = \left[\frac{H_j}{3}\right], \tag{3}$$

where W_j and H_j are the width and height of the warehouse D_j, $j = 1, \ldots, m$, and $[z]$ denotes the integer part of the real number z. Then, for each warehouse denoted by D_j we can compute a specific parameter called *Total_TEU_Length* ($T(j)$) given by

$$T(j) = L(j) \cdot C(j) \cdot R(J) \tag{4}$$

where $L(j)$ is the length of the warehouse D_j, and $R(j), C(j)$ are computed as in (3). More clear - $T(j)$ represents the maximum length of a series of containers that can be stored in D_j, expressed in TEU, assuming that they were placed in a straight line.
Let us suppose that there exist 7 sources of containers $S_1, \ldots, S_7$, each source S_i providing **only containers of** $Type(i)$, $i = 1, \ldots, 7$ (see Table 2.1). We will denote by x_{ij} the number of containers of $Type(i)$ (i.e. from source S_i) which will be transported to the warehouse D_j. Let us suppose that the dimensions of the warehouses $D_1, \ldots, D_7$ are those from Table 2.3 below.
Hence, we can compute $R(j)$ and $C(j)$ according to (3) and get the values from Table 3.1. Then, according to the specific lengths of the containers from each source $S_1, \ldots, S_7$ (see column 2 on Table 2.1), the restrictions (inequalities) imposed by the dimensions of each warehouse can be written as follows

Table 3: The dimensions of the warehouses

Characteristic / Warehouses	D_1	D_2	D_3	D_4	D_5	D_6	D_7
Length [L] (m)	125	78	87	95	72	65	85
Width [W] (m)	40	24	35	47	60	65	72
Height [H] (m)	15	16	14	19	20	17	18

$$\sum_{i=1}^{7} l_i x_{ij} \leq T(j), j = 1, \ldots, 7, \tag{5}$$

where $l = (6.1, 12.2, 13.7, 14.6, 16.2, 6.1, 6.1)$ is a vector containing the lengths in meters from the second column of Table 2.1.

Table 4: The maximum number of rows and columns for each warehouse

Warehouse	D_1	D_2	D_3	D_4	D_5	D_6	D_7
R	13	8	11	15	20	21	24
C	5	5	4	6	6	5	6

To these inequalities we must add the supply/demand restrictions from (1). For this, we will consider the unbalanced and inconsistent transport problem described in Table 2.2 for which the inequalities from (1) are the following

Table 5: The unbalanced transportation problem

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	Supply(s)
S_1	3	3	4	12	20	5	9	1050
S_2	7	1	5	3	6	8	4	350
S_3	5	4	7	6	5	12	3	470
S_4	4	5	14	10	9	8	7	600
S_5	8	2	12	9	8	4	2	600
S_6	6	1	8	7	2	3	1	480
S_7	9	10	6	8	7	6	5	450
Demand(d)	455	320	540	460	760	830	780	

$$\sum_{i=1}^{7} x_{ij} \geq d_j, j = 1, \ldots, 7, \tag{6}$$

$$\sum_{j=1}^{7} x_{ij} = s_i, i = 1, \ldots, 7. \tag{7}$$

But, in our maritime container transportation model problem x_{ij} represents the number of containers of $Type(i)$ transported to the warehouse D_j, and **not** the numbers of TEU's. Hence, we have to introduce in the relations (6)-(7) the weights

$T_w = (1, 2, 2.25, 2.4, 2.65, 1, 1)$ from the last column of Table 2.1 and we obtain

$$\sum_{i=1}^{7} T_{w_i} x_{ij} \geq d_j, \; j = 1, \ldots, 7, \tag{8}$$

$$T_{w_i} \sum_{j=1}^{7} x_{ij} = s_i, \; i = 1, \ldots, 7. \tag{9}$$

Likewise, we have to remodel the matrix of shipping cost C from Table 2.2, by multiplying its entries with the corresponding weights $T_{w_i}, i = 1, \ldots, 7$ from the last column of Table 2.1, and we get the values from Table 6.

Table 6: Matrix C of shipping costs for containers transportation problem

3	3	4	12	20	5	9
14	2	10	6	12	16	8
11.25	9	15.75	13.5	11.25	27	6.75
9.6	12	33.6	24	21.6	19.2	16.8
21.2	5.3	31.8	23.85	21.2	10.6	5.3
6	1	8	7	2	3	1
9	10	6	8	7	6	5

3. NUMERICAL EXPERIMENTS

We will present in this section some numerical experiments on inconsistent transportation problems as those described in section 2. They are performed with the Matlab R2010a *linprog* implementation of Simplex solver and the Modified Han algorithm proposed by us in [2]. In this respect we will prove in what follows a result mentioned (without proof) by Han in his original paper [4]. This result gives us the possibility to express in an equivalent way the primal-dual pair of linear programs

$$\min\langle c, y\rangle \;\text{ subject to }\; By \geq d, \;\; y \geq 0, \tag{10}$$

$$\max\langle d, u\rangle \;\text{ subject to }\; B^T u \leq c, \;\; u \geq 0, \tag{11}$$

$B : m \times n, c, y \in \boldsymbol{R}^n, d, u \in \boldsymbol{R}^m$, as a linear system of inequalities ($\langle\cdot,\cdot\rangle$, $\|\cdot\|$ denote the Euclidean scalar product and norm, respectively). We will denote by $\mathcal{P}, \mathcal{D}$ the set of feasible solutions of the primal (10) and dual (11) problem, respectively.

Proposition 1. *Let us suppose that both problems (10)-(11) have feasible solutions, i.e.* $\mathcal{P} \neq \emptyset, \mathcal{D} \neq \emptyset$*. Then the following assumptions are equivalent:*
(i) $\hat{y} \in \mathcal{P}, \hat{u} \in \mathcal{D}$ *are optimal solutions for problems* $\mathcal{P}$ *and* $\mathcal{D}$*, respectively.*
(ii) the vector $x = [\hat{y}^T, \hat{u}^T]^T \in \boldsymbol{R}^{m \times n}$ *is a solution of the system of linear inequalities*

$$Ax \leq b, \tag{12}$$

where

$$A = \begin{bmatrix} c^T & -d^T \\ -B & 0 \\ 0 & B^T \\ -I & 0 \\ 0 & -I \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -d \\ c \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

Proof. **(i)** $\Rightarrow$ **(ii)** If $\hat{y}, \hat{u}$ are optimal solutions for (10) and (11), respectively, then $\hat{y} \in \mathcal{P}, \hat{u} \in \mathcal{D}$ and (see [1], Corollary 5.1, page 182)

$$\langle c, \hat{y} \rangle = \langle d, \hat{u} \rangle. \tag{14}$$

All these give us the fact that $x = [\hat{y}^T, \hat{u}^T]^T$ satisfies (12) - (13).
(ii) $\Rightarrow$ **(i)** If $x = [\hat{y}^T, \hat{u}^T]^T$ is a solution of (12) - (13), by writing in details the inequality $Ax - b \leq 0$ we obtain that $\hat{y} \in \mathcal{P}, \hat{u} \in \mathcal{D}$ and

$$\langle c, \hat{y} \rangle - \langle d, \hat{u} \rangle \leq 0. \tag{15}$$

But, because $\hat{y}^T, \hat{u}^T$ are feasible solutions, from [1], Proposition 5.1, page 180 it results $\langle c, \hat{y} \rangle \geq \langle d, \hat{u} \rangle$, which together with (15) gives us (14). Then, according to Proposition 5.2, page 181 in [1] $\hat{y}^T, \hat{u}^T$ are optimal solutions for (10) and (11), respectively, and the proof is complete. ∎

Hence, from the above result it holds that solving a pair of *feasible* primal-dual linear programs is equivalent with solving a certain system of linear inequalities. According to Theorem 5.1, page 181 in [1], there are two more possible cases that can occur beside the feasible one: one of the problem has feasible solutions and the other does not, and both problems do not have feasible solutions. In these cases, as Han himself mentioned in [4], the system (12) - (13) provides a kind of *least squares solution* for one or both linear programs, respectively. Such a situation will be considered in the rest of the section.
According to the considerations and constructions in section 2 above, we will analyse in our numerical experiments the following two problems:
Problem P1 - the unbalanced inconsistent problem (1), with the costs c_{ij} from Table 2.2, and the restrictions $(*) - (**)$ corresponding to the relations (6)-(7).
Problem P2 - the inconsistent maritime container transportation problem of the form (1), with the costs $\hat{c}_{ij}$ from Table 6, the restrictions $(*)$ corresponding to the relations

(5) and (8), and the restrictions (∗∗) corresponding to the relation (9).
In this respect, we first renumber the unknowns as

$$x_{ij} \to y_l, i \in \{1, \dots 7\}, j \in \{1, \dots 7\}, l \in \{1, \dots 49\} \tag{16}$$

Let $c, \hat{c} \in \boldsymbol{R}^{49}$ be the cost vectors of the above problems, B_1, B_2 the 7×49 matrices corresponding to the 7 inequalities from (6) and the 7 equalities from (7), respectively, and $\hat{B}_0, \hat{B}_1, \hat{B}_2$ the 7×49 matrices corresponding to the 7 inequalities in (5), the 7 inequalities in (8) and the 7 equalities in (9), respectively, (all of them constructed according to the renumbering (16)). Moreover, let $T, d, s \in \boldsymbol{R}^7$ be defined by (see (4) and Table 2.2):

$$T = (8125, 3120, 3828, 8550, 8640, 6825, 12240)^T,$$

$$d = (455, 320, 540, 460, 760, 830, 780)^T, \; s = (1050, 350, 470, 600, 600, 480, 450)^T. \tag{17}$$

Then, the above problems **P1, P2** can be written as follows
Problem P1

$$\min\langle c, y\rangle \;\text{ s.t. }\; B_1 y \geq d, \;\; B_2 y = s, \;\; y \geq 0. \tag{18}$$

Problem P2

$$\min\langle \hat{c}, y\rangle \;\text{ s.t. }\; \hat{B}_0 y \leq T, \;\; \hat{B}_1 y \geq d, \;\; \hat{B}_2 y = s, \;\; y \geq 0. \tag{19}$$

If we define the matrices $B : 21 \times 49$, $\hat{B} : 28 \times 49$ by

$$B = [B_1^T B_2^T - B_2^T]^T, \quad \hat{B} = [-\hat{B}_0^T \hat{B}_1^T \hat{B}_2^T - \hat{B}_2^T]^T \tag{20}$$

and the vectors $d \in \boldsymbol{R}^{21}$, $\hat{d} \in \boldsymbol{R}^{28}$ by

$$d = [d^T s^T - s^T], \quad \hat{d} = [-T^T d^T s^T - s^T] \tag{21}$$

the problems **P1** (18), and **P2** (19) can be written as

$$\min\langle c, y\rangle \;\text{ s.t. }\; By \geq d, \;\; y \geq 0, \tag{22}$$

$$\min\langle \hat{c}, y\rangle \;\text{ s.t. }\; \hat{B} y \geq \hat{d}, \;\; y \geq 0, \tag{23}$$

with the corresponding dual problems given by (see e.g. [9])

$$\max\langle d, u\rangle \;\text{ s.t. }\; B^T u \leq c, \;\; u \geq 0, \tag{24}$$

$$\max\langle \hat{d}, u\rangle \;\text{ s.t. }\; \hat{B}^T u \leq \hat{c}, \;\; u \geq 0, \tag{25}$$

respectively.
According to the discussion from the beginning of this section, solving the pair of dual problems (22)-(24) and (23)-(25) is equivalent with solving the systems of inequalities $Ax \leq b$ and $\hat{A}\hat{x} \leq \hat{b}$, respectively, where $x = [y^T, u^T]^T \in \boldsymbol{R}^{49} \times \boldsymbol{R}^{21}$ and

$\hat{x} = [\hat{y}^T, \hat{u}^T]^T \in \boldsymbol{R}^{49} \times \boldsymbol{R}^{28}$, and $A, b, \hat{A}, \hat{b}$ are constructed as in (13).
Our transportation problems **P1** and **P2**, together with their duals, are inconsistent, so will be the systems of linear inequalities $Ax \leq b$ and $\hat{A}\hat{x} \leq \hat{b}$. The problems were solved with *linprog* Matlab implementation of Simplex algorithm, whereas the two associated systems with Modified Han (MH) algorithm presented below (see for details [2]):
Let **ALG** be an iterative algorithm which approximates the minimal norm solution of a linear least squares problem of the form

$$\| Bx - c \| = \min! \tag{26}$$

where B is an arbitrary rectangular matrix and c an appropriate vector.
Algorithm MH. *Let $x^0 \in \boldsymbol{R}^n$ be arbitrary fixed; for $k = 0, 1, \ldots$ do:*
Step 1. *Find $I_k = I(x^k)$ and compute an approximation $d^{k,j} \in \boldsymbol{R}^n$ of the minimal norm solution of the linear equalities least squares problem*

$$\| A_{I_k} d - (b_{I_k} - A_{I_k} x^k) \| = \min! \tag{27}$$

by performing $j \geq 1$ iterations of the algorithm **ALG***, with* 0 *as initial approximation on (27).*
Step 2. *Compute $\lambda^{k,j} \in \boldsymbol{R}$ as the smallest minimizer of $\theta(\lambda) = f(x^k + \lambda d^{k,j}), \lambda \in \boldsymbol{R}$.*
Step 3. *Set $x^{k+1} = x^k + \lambda^{k,j} d^{k,j}$.*
As **ALG** in Step 1 of the algorithm MH we used the Kaczmarz Extended (KE) algorithm from [6].
In our computations implemented in Matlab R2010a, all runs with respect to **MH** algorithm are started with the initial approximations $x_0 = (y_0^T, 0)^T$, $\hat{x}_0 = (\hat{y}_0^T, 0)^T$, with $y_0 \geq 0, \hat{y}_0 \geq 0$, and are terminated if at the current iterations $x^k, \hat{x}^k$ satisfy

$$\| A^T(Ax^k - b)_+ \| \leq 10^{-14}, \quad \| \hat{A}^T(\hat{A}\hat{x}^k - \hat{b})_+ \| \leq 10^{-14}, \tag{28}$$

where $z \in \boldsymbol{R}^n$, $z_+ \in \boldsymbol{R}^n$ denote the vector with components $(z_+)_i = z_i$, if $z_i \geq 0$ and $(z_+)_i = 0$, if $z_i < 0$. Results are presented in Tables 7 and 8.

Remark 2. *The expressions from (28) are related to some theoretical properties of original Han's algorithm from [4] when solving a system of linear inequalities of the form (12). More clear, Han proves that $x^* \in \boldsymbol{R}^n$ is a solution of (12) if and only if it satisfies the normal equation in the inequality case $A^T(Ax - b)_+ = 0$ (some details can be found in [3]).*

where * denotes that the Simplex algorithm failed to solve the problem, returning instead a result that minimizes *the worst case constraint violation* (see [9]).
Let $w = Fy - f \in \boldsymbol{R}^{14}$ where $F = [B_1^T B_2^T]^T$, $f = [d_1^T d_2^T]^T$ and $\hat{w} = \hat{F}\hat{y} - \hat{f} \in \boldsymbol{R}^{21}$ where $\hat{F} = [\hat{B}_0^T \hat{B}_1^T \hat{B}_2^T]^T$, $\hat{f} = [T^T d^T s^T]^T$ be the vectors of the unmet inequalities

Algorithm	cost	$\parallel (Ax-b)_+ \parallel$
MH	15336	38.7529
Simplex	31235*	145.0241

Table 7: Results for problem P1

Table 8: Results for problem P2

Algorithm	cost	$\parallel (\hat{A}x-\hat{b})_+ \parallel$
MH	15336	38.7529
Simplex	29070*	145.0626

Table 9: The values of unmet inequalities and equalities for problem **P1**

w	MH	Simplex
w_1	-10	0
w_2	-10	0
w_3	-10	0
w_4	-10	-145
w_5	-10	0
w_6	-10	0
w_7	-10	0
w_8	10	0
w_9	10	0
w_{10}	10	0
w_{11}	10	0
w_{12}	10	0
w_{13}	10	0
w_{14}	10	0

Table 10: The values of unmet inequalities and equalities for problem **P2**

$\hat{w}$	MH	Simplex
$\hat{w}_1$	-5420	-5349
$\hat{w}_2$	-1233	-1168
$\hat{w}_3$	-597	-534
$\hat{w}_4$	-5808	-5744
$\hat{w}_5$	-4069	-4006
$\hat{w}_6$	-1825	-1774
$\hat{w}_7$	-5498	-6319
$\hat{w}_8$	-10	0
$\hat{w}_9$	-10	0
$\hat{w}_{10}$	-10	0
$\hat{w}_{11}$	-10	0
$\hat{w}_{12}$	-10	0
$\hat{w}_{13}$	-10	0
$\hat{w}_{14}$	-10	-145
$\hat{w}_{15}$	10	0
$\hat{w}_{16}$	10	0
$\hat{w}_{17}$	10	0
$\hat{w}_{18}$	10	0
$\hat{w}_{19}$	10	0
$\hat{w}_{20}$	10	0
$\hat{w}_{21}$	10	0

and equalities (6)-(7) and (5), (8)-(9), respectively. The values of the components of vectors w and $\hat{w}$ for Simplex and MH algorithms are presented in Tables 9 and 10.

Tables 11 and 12 indicate the solutions obtained for the inconsistent transportation problem **P1**. We observe that MH algorithm solution is more reliable (for a practical view point).

4. CONCLUSIONS

In this paper we first considered an inconsistent version of the classical transportation problem (**Problem P1**). Based on its transportation (inconsistent) assumption, we derive the maritime container transportation problem (**Problem P2**). We propose

Table 11: The values $x_{ij}, i = 1, \ldots, 7, j = 1, \ldots, 7$ for the solution of problem **P1** with MH algorithm.

i/j	1	2	3	4	5	6	7
1	0	144	530	0	0	387	0
2	0	0	0	360	0	0	0
3	0	0	0	89	232	0	159
4	445	166	0	0	0	0	0
5	0	0	0	0	0	0	610
6	0	0	0	0	490	0	0
7	0	0	0	0	28	433	0

Table 12: The values $x_{ij}, i = 1, \ldots, 7, j = 1, \ldots, 7$ for the solution of problem **P1** with Simplex algorithm.

i/j	1	2	3	4	5	6	7
1	0	0	0	0	0	415	635
2	0	0	0	0	0	350	0
3	0	0	0	0	405	65	0
4	0	0	0	245	355	0	0
5	0	0	385	215	0	0	0
6	0	320	155	0	0	0	0
7	450	0	0	0	0	0	0

a mathematical model for it (Section 2), then we solve both problems **P1** and **P2** by the Modified Han iterative algorithm from [2] and compare the results with those provided by the Matlab *linprog* routine. We conclude that our approach produces more reliable results than the classical Simplex-like methods (see Tables 7-8 and 11-12). As a further step in our research, we are interested in eliminating the assumption (see Section 2.2) constraining each source S_i to provide only one category of containers ($Type(i)$). Work is in project on this subject.

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