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ASYMPTOTIC EQUATIONS OF THE NONLINEAR TRANSONIC GAS FLOWS AND THEIR SOLUTIONS

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Abstract In the article there is received an equation for the transonic gas flows which considers transversals disturbance surpassing the main flow disturbance. Some exact individual solutions of this equation are shown as well as their application to the solutions of a number of tasks.

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1. DEDUCTION OF THE ASYMPTOTIC EQUATION

Irrotational isentropic gas flows in the cylindrical dimensionless coordinates x, r, θ are being described with the help of the equation:

$$\begin{gathered}\Phi_{tt} + 2\Phi_x\Phi_{xt} + 2\Phi_r\Phi_{rt} + \frac{2}{r^2}\Phi_\theta\Phi_{\theta t} + 2\Phi_x\Phi_r\Phi_{rx} + \frac{2}{r^2}\Phi_x\Phi_\theta\Phi_{\theta x} + \\ + \frac{2}{r^2}\Phi_\theta\Phi_r\Phi_{\theta r} + \Phi_x^2\Phi_{xx} + \Phi_r^2\Phi_{rr} + \frac{1}{r^4}\Phi_\theta^2\Phi_{\theta\theta} - \\ -a^2\left(\Phi_{xx} + \Phi_{rr} + \frac{1}{r}\Phi_r + \frac{1}{r^2}\Phi_{\theta\theta}\right) = 0, \\ a^2 = \rho^{\chi-1} = p^{\frac{\chi-1}{\chi}} = \frac{\chi+1}{2} - \frac{\chi-1}{2}\left(2\Phi_t + \Phi_x^2 + \Phi_r^2 + \frac{1}{2r^2}\Phi_\theta^2\right).\end{gathered} \tag{1}$$

In(1) $\Phi(x, r, \theta, t)$ means the potential of speed, t is the time, a is the sound speed, ρ is the density, p is the pressure. Let us put into an asymptotic decomposition for $\Phi(x, r, \theta, t)$

$$\Phi = x + \epsilon\psi(r, \theta, t^0) + \epsilon^3\varphi(x^0, r, \theta, t^0) + ..., \ x = \epsilon x^0, \ t = \frac{1}{\epsilon}t^0, \tag{2}$$

where ϵ is a small parameter. If we substitute (2) in (1) and leave the part of the main order aside, we will get a transonic equation for the function $\varphi(x^0, r, \theta, t^0)$:

$$\begin{gathered}2\varphi_{x^0t^0} + (\chi+1)\varphi_{x^0}\varphi_{x^0x^0} + 2\psi_r\varphi_{x^0r} + \frac{2}{r^2}\psi_\theta\varphi_{x^0\theta} + \\ + \frac{\chi-1}{2}\left(2\psi_{t^0} + \psi_r^2 + \frac{1}{r^2}\psi_\theta^2\right)\varphi_{x^0x^0} - \Delta\varphi = L(\psi).\end{gathered} \tag{3}$$

In (3) are put into the following designations

$$\begin{gathered}\Delta\varphi \equiv \varphi_{rr} + \frac{1}{r}\varphi_r + \frac{1}{r^2}\varphi_{\theta\theta}, \\ -L(\psi) \equiv \psi_{t^0t^0} + 2\psi_r\psi_{rt^0} + \frac{2}{r^2}\psi_\theta\psi_{\theta t^0} + \psi_r^2\psi_{rr} + \frac{1}{r^4}\psi_\theta^2\psi_{\theta\theta} + \\ + \frac{2}{r^2}\psi_\theta\psi_r\psi_{r\theta} - \frac{1}{r^3}\psi_r\psi_\theta^2.\end{gathered}$$

The function $\psi(r, \theta, t^0)$ satisfies the Laplace's equation $\Delta\psi = 0$. In case $\psi \equiv 0$ we get a classical transonic Lin-Raissner-Tzyan's equation

$$2\varphi_{x^0t^0} + (\chi+1)\varphi_{x^0}\varphi_{x^0x^0} - \varphi_{rr} - \frac{1}{r}\varphi_r - \frac{1}{r^2}\varphi_{\theta\theta} = 0,$$

which proceeds to the Karman-Facovitch's mixed type equation in permanent case: $(\chi+1)\varphi_{x^0}\varphi_{x^0x^0} - \varphi_{rr} - \frac{1}{r}\varphi_r - \frac{1}{r^2}\varphi_{\theta\theta} = 0$.

The equation (3) describes transonic gas flows, which appear because of the influence on the streamline solid the side (with respect to the main stream course which coincides with the axis x) disturbance of the main transonic flow (for the disturbing cross flow $\Phi_y, \Phi_z \sim \epsilon$, for the main flow $\Phi_y, \Phi_z \sim \epsilon^3$). The disturbance for the external streamlines of the aircrafts is, for instance, the side wind, which changes its intensity depending on time $\psi = V_\infty(t) r \cos(\theta + \alpha(t))$. For the internal streamline, for example, the disturbance can be a flow twist ($\psi = \Gamma(t)\theta$).

We can get the conditions at the front a shock wave $x^0 = x^0(r, \theta, t^0)$ from the Renckin-Gyugonio conditions, substituting the decomposition (2) in them and leaving the main order parts aside:

$$\begin{gathered}2\frac{\partial x^0}{\partial t^0} + \left(\frac{\partial x^0}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial x^0}{\partial \theta}\right)^2 + 2\psi_r\frac{\partial x^0}{\partial r} + \frac{2}{r^2}\psi_\theta\frac{\partial x^0}{\partial \theta} = \\ = \frac{\chi-1}{2}\left(2\psi_{t^0} + \psi_r^2 + \frac{1}{r^2}\psi_\theta^2\right) + \frac{\chi+1}{2}(\varphi_{x^0} + \varphi^*_{x^0}), \quad \varphi = \varphi^*.\end{gathered} \tag{4}$$

If we put $\varphi \equiv \varphi^*$ in (4), we will get a defining equation for (3).

Let us write down the conditions on the streamline surface, which differs from the cylindrical surface just a little bit, giving it as

$$r = r_0(\theta, t^0) + r_2(x^0, \theta, t^0)\epsilon^4 + ... \tag{5}$$

Substituting (2) and (5) into the exact non-passing condition

$$-\Phi_x r_x + \Phi_r - r^{-2} r_\theta \Phi_\theta = r_t$$

and leaving the main order parts aside, then we get:

$$\psi_r - \frac{1}{r_0^2}\frac{\partial r_0}{\partial \theta}\psi_\theta = \frac{\partial r_0}{\partial t^0}, \quad \varphi_r - \frac{1}{r_0^2}\frac{\partial r_0}{\partial \theta}\varphi_\theta = \frac{\partial r_2}{\partial x^0}. \tag{6}$$

The values of φ_r, φ_θ, ψ_r, ψ_θ in (6) is being calculated when $r = r_0(\theta, t^0)$.

The sound surface equation ($V^2 = a^2$) in transonic approximation takes the form

$$N \equiv \frac{\chi + 1}{2}\left(\psi_r^2 + \frac{1}{r^2}\psi_\theta^2\right) + (\chi - 1)\psi_{t^0} + (\chi + 1)\varphi_{x^0} = 0. \tag{7}$$

For steady flow the equation (3) has the mixed type.

Substituting (2) into the expression for the pressure, conducting the decomposition into the Taylor's series and leaving the main order parts aside, we get an asymptotic formula for the pressure definition

$$P = 1 - \chi\epsilon^2\left(\psi_{t^0} + \varphi_{x^0} + \frac{1}{2}\psi_r^2 + \frac{1}{2r^2}\psi_\theta^2\right). \tag{8}$$

2. SOME OF THE SOLUTIONS OF THE ASYMPTOTIC EQUATION (3)

Let us indicate some of the partial solutions of the equation (3).

Let us mark **the automodel class** of the solutions (indices zero of variables x, t is left out here and further):

$$\psi = t^\beta \overline{\psi}(\zeta, \eta), \ \varphi = t^{2\beta - 1}\overline{\varphi}(\xi, \zeta, \eta), \ \xi = \frac{x}{t^\beta}, \ \zeta = \frac{r}{t^{(\beta+1)/2}}, \ \eta = \theta + \alpha \ln t, \tag{9}$$

where α, β are arbitrary numbers. After substitution (9) into the equation (3), we will get a equation for function $\varphi = \overline{\varphi}(\xi, \zeta, \eta)$

$$
\begin{gathered}
2\left((\beta-1)\overline{\varphi}_{\xi} - \beta\overline{\varphi}_{\xi\xi}\xi - \frac{\beta+1}{2}\overline{\varphi}_{\xi\zeta} + \alpha\overline{\varphi}_{\xi\eta}\right) + \\
+(\chi+1)\overline{\varphi}_{\xi}\overline{\varphi}_{\xi\xi} + 2\overline{\psi}_{\zeta}\overline{\varphi}_{\xi\zeta} + \frac{2}{\zeta^2}\overline{\psi}_{\eta}\overline{\varphi}_{\xi\eta} - \Delta\overline{\varphi} + \\
+\frac{\chi-1}{2}\left(2\beta\overline{\psi} - (\beta+1)\zeta\overline{\psi}_{\zeta} + 2\alpha\overline{\psi}_{\eta} + \overline{\psi}_{\zeta}^2 + \frac{1}{\zeta^2}\overline{\psi}_{\eta}^2\right)\overline{\varphi}_{\xi\xi} = L(\overline{\psi}), \\
\Delta\overline{\varphi} = \overline{\varphi}_{\zeta\zeta} + \frac{1}{\zeta}\overline{\varphi}_{\zeta} + \frac{1}{\zeta^2}\overline{\varphi}_{\eta\eta}, \\
-L(\overline{\psi}) = \beta(\beta-1)\overline{\psi} + \beta\left(-\frac{\beta+1}{2}\zeta\overline{\psi}_{\zeta} + \alpha\overline{\psi}_{\eta}\right) + (\beta-1)\left(-\frac{\beta+1}{2}\zeta\overline{\psi}_{\zeta} + \right. \\
\left. +\alpha\overline{\psi}_{\eta}\right) + \left(\frac{\beta+1}{2}\right)^2\zeta^2\overline{\psi}_{\zeta\zeta} + \alpha^2\overline{\psi}_{\eta\eta} + 2\overline{\psi}_{\zeta}\left(\beta\overline{\psi}_{\zeta} - \frac{\beta+1}{2}\zeta\overline{\psi}_{\zeta\zeta} + \alpha\overline{\psi}_{\eta\zeta}\right) + \\
+\frac{2}{\zeta^2}\overline{\psi}_{\eta}\overline{\psi}_{\zeta}\overline{\psi}_{\eta\zeta} + \frac{2}{\zeta^2}\overline{\psi}_{\eta}\left(\beta\overline{\psi}_{\eta} - \frac{\beta+1}{2}\zeta\overline{\psi}_{\zeta\eta} + \alpha\overline{\psi}_{\eta}\right) + \overline{\psi}_{\zeta}^2\overline{\psi}_{\zeta\zeta} + \frac{1}{\zeta^4}\overline{\psi}_{\eta}^2\overline{\psi}_{\eta\eta} - \\
-\frac{1}{\zeta^3}\overline{\psi}_{\zeta}\overline{\psi}_{\eta}^2.
\end{gathered}
$$

Function $\overline{\psi}$ satisfies the Laplace's equation: $\overline{\psi}_{\zeta\zeta} + \frac{1}{\zeta}\overline{\psi}_{\zeta} + \frac{1}{\zeta^2}\overline{\psi}_{\eta\eta} = 0$.

Equation (3) permits **polynomial type** of solution

$$
\varphi = \sum_{k=0}^{3} \varphi_k(r, \theta, t)x^k. \tag{10}
$$

After substitution (10) into the equation (3) we will get a system of equations for functions $\varphi_0(r, \theta, t)$, $\varphi_1(r, \theta, t)$, $\varphi_2(r, \theta, t)$, $\varphi_3(r, \theta, t)$:

$$
\begin{cases}
18\varphi_3^2(\chi+1) - \Delta\varphi_3 = 0, \\
6\varphi_{3t} + 18(\chi+1)\varphi_2\varphi_3 + 6\psi_r\varphi_{3r} + \frac{6}{r^2}\psi_\theta\varphi_{3\theta} - \Delta\varphi_2 = 0, \\
4\varphi_{2t} + (\chi+1)(6\varphi_1\varphi_3 + 4\varphi_2^2) + 4\psi_r\varphi_{2r} + \frac{4}{r^2}\psi_\theta\varphi_{2\theta} - \Delta\varphi_1 + \\
+6(\chi-1)\varphi_3\left(\psi_t + \frac{1}{2}\psi_r^2 + \frac{1}{2r^2}\psi_\theta^2\right) = 0, \\
2\varphi_{1t} + 2(\chi+1)\varphi_1\varphi_2 + 2\psi_r\varphi_{1r} + \frac{2}{r^2}\psi_\theta\varphi_{1\theta} - \Delta\varphi_0 + \\
+2(\chi-1)\varphi_2\left(\psi_t + \frac{1}{2}\psi_r^2 + \frac{1}{2r^2}\psi_\theta^2\right) = -L(\psi).
\end{cases}
$$

In the class of solutions (10) there is a solution in case of the steady flows, which describes the gas flow in the Lavales's nozzles with the permanent acceleration

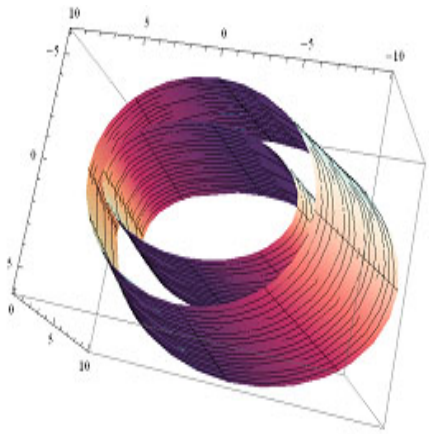

Fig. 1.: The streamlined surface of the form (12) at $r_0 = 3$, $r_0 = 5$.

($\varphi_{xx} = const$) and considers the flow twist ($\psi = \Gamma\theta$, $\Gamma = const$):

$$\varphi = ax^2 + (\chi + 1)a^2r^2x + \left[\frac{\chi - 1}{2}\Gamma^2 a \ln^2 r + \frac{1}{8}(\chi + 1)^2 a^3 r^4\right]. \tag{11}$$

If $r \to 0$, the speed constituent $V_r = \epsilon^3\varphi_r$ has a peculiarity $\ln r/r$, which is conditioned by the primordial peculiarity of the constituent assignment $V_\theta(V_\theta = \epsilon\Gamma/r)$. The sound surface equation for (11), according to (7), takes the form

$$x = -\frac{\chi + 1}{2}ar^2 - \frac{\Gamma^2}{4ar^2}.$$

If $\chi \to 1$, the influence of the flow twist on the speed distribution gets smaller and the peculiarity for φ_r disappears if $r \to 0$. Conditions (6) take a form: $\dfrac{\partial r_0}{\partial\theta} = 0$, $\dfrac{\partial r_2}{\partial x} = \varphi_r$ (φ_r is being calculated at $r = r_0(\theta)$). Defining functions $r_0(\theta)$ and $r_2(x, \theta)$, we will get a streamline surface equation:

$$r = \epsilon^4\left[r_0a^2(\chi + 1)x^2 + \left((\chi - 1)\Gamma^2 a\frac{\ln r_0}{r_0} + \frac{1}{2}(\chi + 1)^2a^3r_0^3\right)x + \right. \\ \left. + C(\theta)\right] + r_0,\ r_0 = const. \tag{12}$$

The solution (11) can be used for describing the flows in the annular channel, the equations of the internal and external sides can be gotten from (12) at $r_0 = r_0^{(1)}$, $r_0 = r_0^{(2)}$, $r_0^{(k)} = const \neq 0$ (see fig.1).

In case $\Gamma = 0$ in (11) we will get a known solution that describes the flow in the center of the Lavales's nozzle. Let us define the solution for cases $\psi = \Gamma\theta$ that has features: $V_x,\ V_r,\ V_\theta \to 0$ at $r \to \infty$

$$\varphi = f(x, \theta)r^{-2} + g(r, \theta),\ \Delta g = 0. \tag{13}$$

Substituting the solution (13) and ψ into the equation (3), we will get an equation for the function $f(x,\theta)$:

$$(\chi+1)f_x f_{xx} + 2\Gamma f_{x\theta} + (\chi-1)\Gamma^2 f_{xx} - 4f - f_{\theta\theta} = 0.$$

The solution (13) permits the generalization: $\varphi = f(\xi,\eta)r^{-2} + g(r,\theta)$, $\xi = x + \alpha \ln r$, $\eta = \theta + \beta \ln r$, α, β are arbitrary numbers.

Let us examine the case, corresponding **the gas flow between revolving flatnesses** $\theta = \theta_1(t)$, $\theta = \theta_2(t)$. In this case

$$\psi = r^2(a(t)\cos 2\theta + b(t)\sin 2\theta) = r^2 f(\theta,t), \tag{14}$$

at the same time in (3)

$$\begin{gathered} L(\psi) = r^2(8(a'a + b'b) + 8(a\cos 2\theta + b\sin 2\theta)(2ab\sin 4\theta + \\ +a^2\cos 4\theta - b^2\cos 4\theta) + a''\cos 2\theta + b''\sin 2\theta) = r^2 G(\theta,t). \end{gathered}$$

Functions $a(t)$, $b(t)$ are being defined from the conditions of non-passing (6):

$$f_\theta(\theta_k(t),t) = \frac{\partial\theta_k(t)}{\partial t},\ k = 1,2,$$

and are equal

$$a(t) = \frac{1}{A}(\theta_2'\cos 2\theta_2 - \theta_1'\cos 2\theta_1),\ b(t) = \frac{1}{A}(\theta_2'\sin 2\theta_1 - \theta_1'\sin 2\theta_2),$$

where $A = 2\sin 2(\theta_1 - \theta_2)$. Substituting (14) into (3), we will get the equation for $\varphi(x,r,\theta,t)$:

$$\begin{gathered} 2\varphi_{xt} + (\chi+1)\varphi_x\varphi_{xx} + 4r(a\cos 2\theta + b\sin 2\theta)\varphi_{xr} + 4(b\cos 2\theta - \\ -a\sin 2\theta)\varphi_{x\theta} - \Delta\varphi + (\chi-1)r^2(a'\cos 2\theta + b'\sin 2\theta + 2a^2 + \\ +2b^2)\varphi_{xx} = -r^2 G(\theta,t). \end{gathered} \tag{15}$$

The equation (15) permits the solution that takes a form of (10). Then we will get a system of four equations for the functions $\varphi_0(r,\theta,t)$, $\varphi_1(r,\theta,t)$, $\varphi_2(r,\theta,t)$, $\varphi_3(r,\theta,t)$:

$$\begin{cases} 18\varphi_3^2(\chi+1) - \Delta\varphi_3 = 0, \\ 6\varphi_{3t} + 18(\chi+1)\varphi_2\varphi_3 + 12(b\cos 2\theta - a\sin 2\theta)\varphi_{3\theta} + \\ +12r(a\cos 2\theta + b\sin 2\theta)\varphi_{3r} - \Delta\varphi_2 = 0, \\ 4\varphi_{2t} + (\chi+1)(6\varphi_1\varphi_3 + 4\varphi_2^2) + 8r(a\cos 2\theta + b\sin 2\theta)\varphi_{2r} - \\ -\Delta\varphi_1 + 8(b\cos 2\theta - a\sin 2\theta)\varphi_{2\theta} + 6(\chi-1)r^2\varphi_3(a'\cos 2\theta + \\ +b'\sin 2\theta + 2a^2 + 2b^2) = 0, \\ 2\varphi_{1t} + 2(\chi+1)\varphi_1\varphi_2 + 4r(a\cos 2\theta + b\sin 2\theta)\varphi_{1r} + 4(b\cos 2\theta - \\ -a\sin 2\theta)\varphi_{1\theta} - \Delta\varphi_0 + 2(\chi-1)r^2(a'\cos 2\theta + b'\sin 2\theta + \\ +2a^2 + 2b^2)\varphi_2 = r^2 G(\theta,t). \end{cases} \tag{16}$$

At the same time we can put $\varphi_3 = g(\theta, t)r^{-2}$, where $g(\theta, t)$ is defined from the equation: $18(\chi + 1)g^2 - 4g - g_{\theta\theta} = 0$.

In a particular case, let us consider the solution for the steady flows

$$\varphi(x, r, \theta) = \varphi_2(\theta)x^2 + \varphi_1(\theta)r^2 x + \varphi_0(\theta)r^4. \tag{17}$$

Substituting (17) into the system (16), we will get a system of three equations for the functions $\varphi_2(\theta)$, $\varphi_1(\theta)$, $\varphi_0(\theta)$:

$$\begin{cases} \varphi_2'' = 0, \\ 4(\chi + 1)\varphi_2^2 + 8(b\cos 2\theta - a\sin 2\theta)\varphi_2' - 4\varphi_1 - \varphi_1'' = 0, \\ 2(\chi + 1)\varphi_1\varphi_2 + 4(b\cos 2\theta - a\sin 2\theta)\varphi_1' - 16\varphi_0 - \varphi_0'' + 8(a\cos 2\theta + \\ +b\sin 2\theta)\varphi_1 + 4(\chi - 1)(a^2 + b^2)\varphi_2 = G(\theta). \end{cases}$$

Considering the non-passing conditions $\left(\frac{\partial f}{\partial\theta}|_{\theta=\theta_1,\theta_2} = 0, \frac{\partial\varphi_k}{\partial\theta}|_{\theta=\theta_1} = 0,\right.$ $\left.\frac{\partial\varphi_k}{\partial\theta}|_{\theta=\theta_2} = 0, k = 0, 1, 2\right)$, we will get $\theta_2 - \theta_1 = \frac{\pi n}{2}, n \in N, \varphi_2 = const = C_1$. Then $\varphi_1 = C_1^2(1 + \chi) + C_2\cos 2\theta$, and the function φ_0 is defined from the equation:

$$\begin{gathered} \varphi_0'' + 16\varphi_0 = 2(\chi + 1)C_1(C_1^2(1 + \chi) + C_2\cos 2\theta) - 8C_2\sin 2\theta(b\cos 2\theta - \\ -a\sin 2\theta) + 8(a\cos 2\theta + b\sin 2\theta)(C_1^2(1 + \chi) + C_2\cos 2\theta) + \\ +4(\chi - 1)(a^2 + b^2)C_1 - 8(a\cos 2\theta + b\sin 2\theta)(2ab\sin 4\theta + a^2\cos 4\theta - \\ -b^2\cos 4\theta) = H(\theta). \end{gathered}$$

Let us consider **the streamline surface, which differs from the cylinder just a little bit** ($r_0 = R$). In this case, assuming the cross-flow of the surface is unseparated, we can suggest $\psi = V_\infty\cos\theta(r + R^2r^{-1})$. Then the equation (3) takes the form

$$\begin{gathered} 2\varphi_{xt} + (\chi + 1)\varphi_x\varphi_{xx} + 2V_\infty\cos\theta\left(1 - \frac{R^2}{r^2}\right)\varphi_{xr} - \Delta\varphi - \\ -2V_\infty\frac{\sin\theta}{r}\left(1 + \frac{R^2}{r^2}\right)\varphi_{x\theta} + \frac{\chi - 1}{2}V_\infty^2\varphi_{xx}\left(1 - 2\frac{R^2}{r^2}\cos 2\theta + \right. \\ \left. + \frac{R^4}{r^4}\right) = -2\left(\frac{R^2}{r^3}\right)V_\infty^3\cos\theta\left(1 - \frac{2R^2}{r^2} + \frac{R^4}{r^4} - 4\sin^2\theta\right). \end{gathered} \tag{18}$$

We can free ourselves from the right side ($\alpha(r, \theta)$) of the equation (18) with the help of the new function $\overline{\varphi} = \varphi + g(r, \theta)$, $\Delta g = \alpha(r, \theta)$. Then in a permanent case we will get an equation for the function $\overline{\varphi}$:

$$\begin{gathered} (\chi + 1)\overline{\varphi}_x\overline{\varphi}_{xx} + 2V_\infty\cos\theta\left(1 - \frac{R^2}{r^2}\right)\overline{\varphi}_{xr} - \Delta\overline{\varphi} - 2V_\infty\frac{\sin\theta}{r}\left(1 + \frac{R^2}{r^2}\right)\overline{\varphi}_{x\theta} + \\ + \frac{\chi - 1}{2}V_\infty^2\overline{\varphi}_{xx}\left(1 - 2\frac{R^2}{r^2}\cos 2\theta + \frac{R^4}{r^4}\right) = 0. \end{gathered} \tag{19}$$

The equation (19) has a solution with the type (10) with φ_k depended on r, θ, at the same time we can assume $\varphi_3 = g(\theta)r^{-2}$ (in particular, $g = 0$ or $g = 1/(3(\chi+1)\cos^2\theta)$). For the distant from the solid flow $r \to 0$, the solution for the equation (19) can be being found in the form

$$\overline{\varphi} = r^{\lambda} f(\xi, \theta) + ..., \ \xi = xr^{-n}.$$

We can assume $\lambda = 3n - 2$, because at $V_\infty = 0$ we must get an asymptotic corresponded to the classical equation $(\chi + 1)\varphi_x\varphi_{xx} - \Delta\varphi = 0$. In the case of the formal transition to (18) at $r \to \infty$ we will get a limit equation

$$\begin{gathered} M(\varphi) = 2\varphi_{xt} + (\chi + 1)\varphi_x\varphi_{xx} + 2V_\infty \cos\theta\varphi_{xr} - (2/r)V_\infty \sin\theta\varphi_{x\theta} - \\ -\Delta\varphi + ((\chi - 1)/2)V_\infty^2\varphi_{xx} = -(2R^2/r^3)V_\infty^3 \cos\theta(1 - 4\sin^2\theta), \end{gathered} \tag{20}$$

which permits an exact solution in a permanent case

$$\varphi = rf(\xi, \theta) + g(\theta)r^{-1}, \ \xi = x/r, \ g(\theta) = -R^2V_\infty^3 \sin^2\theta\cos\theta.$$

In case $R \ll 1$, an approximate solution of the equation (18) in a permanent case can be being found in the form

$$\varphi = \sum_{n=0}^{\infty} \widetilde{\varphi}_n(x, r, \theta)R^{2n}.$$

Then for $\widetilde{\varphi}_0$ we will get the equation (20) in a permanent case and without the right side ($M(\widetilde{\varphi}_0) = 0$), which has a solution (10), where φ_n doesn't depend on t, a solution $\widetilde{\varphi}_0 = rf(\xi, \eta) + g(r, \theta)$, $\xi = x/r$, $\eta = \theta + \alpha \ln r$, $\Delta g = 0$. In case $V_\infty \ll 1$ the solution (18) can be being found in the form

$$\varphi = \sum_{n=0}^{\infty} \widetilde{\varphi}_n(x, r, \theta)V_\infty^n.$$

In this case we will can a known transonic equation for $\widetilde{\varphi}_0$

$$(\chi + 1)\widetilde{\varphi}_{0x}\widetilde{\varphi}_{0xx} - \Delta\widetilde{\varphi}_0 = 0.$$

In case of the separation streamline and assuming that two vortical rectilinear shrouds of infinite length with permanent and opposite in sign intensions Γ mare

coming off the surface of the cylinder, the expression for $\psi(r,\theta)$ can be written as:

$$\begin{aligned}\psi(r,\theta) = V_\infty \cos\theta\left(r+\frac{R^2}{r}\right)+\frac{\Gamma}{2\pi}\int\limits_0^\infty\left(arctg\left(\frac{r\sin\theta-R}{r\cos\theta-S}\right)-\right.\\ \left.-arctg\left(\frac{r\sin\theta+R}{r\cos\theta-S}\right)\right)dS+\\ +\frac{\Gamma}{2\pi}\int\limits_0^\infty\left(arctg\left(\frac{r\sin\theta(S^2+R^2)+R^3}{r\cos\theta(S^2+R^2)-SR^2}\right)-\right.\\ \left.-\,arctg\left(\frac{r\sin\theta(S^2+R^2)-R^3}{r\cos\theta(S^2+R^2)-SR^2}\right)\right)dS\end{aligned} \tag{21}$$

The come-off points of the whirlwinds are points $r = R$, $\theta = \pm\frac{\pi}{2}$. According to (8), it is necessary to know $\psi_\theta(R,\theta)$ ($\psi_r(R,\theta) = 0$) in non-passing condition (6), which is satisfied by the function (21) for the definition of the power influence on the streamline solid. Defining ψ_θ from (21) with the help of the θ differentiation and making S integration, we get

$$\begin{aligned}\psi_\theta(R,\theta) = -(\Gamma+2V_\infty)R\sin\theta+\frac{\Gamma R}{2\pi}\left(\cos\theta\ln\left|\frac{1-\sin\theta}{1+\sin\theta}\right|-\right.\\ \left.-2\sin\theta arctg\left(\frac{\cos\theta}{1-\sin\theta}\right)+arctg\left(\frac{\cos\theta}{1+\sin\theta}\right)\right).\end{aligned} \tag{22}$$

The investment into the expression for $C_0 = -\int\limits_0^{2\pi} P\cos\theta d\theta$, corresponding (21)is defined according to (8), (22) by the formula ($V_\infty = R = 1$):

$$\int\limits_0^{2\pi}\psi_\theta^2\cos\theta d\theta = 2\Gamma^2+4\Gamma,\ \Gamma<-2.$$

3. CONCLUSIONS

The article is devoted to the development of the mathematical theory of gas flow with a speed close to the speed of sound, namely transonic gas flows, i.e. flows that contain both subsonic and supersonic area. The main problems arising in the study of such flows should be classified as non-linearity and mixed type equations describing transonic flow. Transonic gas flows taking into account the transverse perturbations are studied on the basis of nonlinear equation obtained in this paper. Some exact particular solutions of this equation are constructed and their application to solving a number of transonic aerodynamics problems are shown. In particular, a solution of polynomial form describing axisymmetric gas flow in Laval nozzles with constant

acceleration and flow swirling is obtained. The unsteady flows in the channels between the rotating planes are researched. The asymptotic equation describing the flows arising at the unseparated and separated flow past the body that is practically identical to the cylindrical one is derived.

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