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EXPLICIT SOLUTIONS OF THE DIFFERENTIAL SYSTEMS AND MATHEMATICAL MODELLING IN THE ELECTROMAGNETISM. PART 1

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Abstract A new analytic method relying on the diagonalization procedure for an arbitrary square system of PDEs (partial differential equations) with piecewise constant coefficients is proposed. Engineering applications are done in terms of the boundary value problems regarding the general wave PDE including all scalar components of the electromagnetic field vector intensities.

The given approach is applied to the differential Maxwell system in the spatial Cartesian coordinates, and it represents the mathematical simulation of the electromagnetic wave propagation in the guided structure under the specific boundary conditions.

Keywords: systems of partial differential equations, differential Maxwell systems, general analytic technique, simulation of the spatial electromagnetic phenomenon.

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1. INTRODUCTION

The present research concerns the development of a new analytic technique regarding the search of the vector-valued field function described by the finite-dimensional system of PDEs (partial differential equations). It is done without generally accepted reduction to the integral equations [1], [2] and deals with the so called "diagonalization procedure" of the original system. The latter is transformed into the equivalent union of equations where each of them depends on the only one scalar component of the initially unknown vector field function.

The diagonalization algorithms are well-known in algebra [3] and in the systems of ODEs (ordinary differential equations)[4], though for the systems of PDEs such apparatus was used only in some particular cases, including the electromagnetic field theory [5].

The proposed here differential operator diagonalization method is generated by the classical algebraic approach of the Gauss procedure [3].

This generalized partial differential operator technique was briefly presented in [6], and was later applied to the various versions of the differential Maxwell systems reducing them to the unified wave PDEs regarding all scalar components of the electromagnetic field vector intensities [7].

Mathematical and computer simulation of the relevant electrodynamic phenomenon was done by the boundary value problem of the aforesaid common wave scalar equation in the specific original statement. Those results are reflected briefly in [8].

Further, while the systems of PDEs with constant coefficients concern the homogeneous medium, the case of the piecewise constant coefficients deals with the heterogeneous one. The latter covers the main requirements of the current industrial problems in the technical electrodynamics. Here, the perfect conductivity σ, the electric and magnetic permeability of the medium ε, μ have just piecewise constant functional behaviour in the differential Maxwell system [9] which remains the basic fundamental mathematical model in the electromagnetic field theory.

Moreover, it should be noted that in general, ε and μ can take also negative values which are very important for using the metamaterials. Nevertheless, relying on the necessary condition following from the electromagnetic equations [10], the electromagnetic wave propagation is possible only in those media whose signs of the electric and magnetic permeability are equal, i.e. $\mathbf{sgn}\ \varepsilon = \mathbf{sgn}\ \mu$ [11] - [13].

In those cases, the reverse waves appear, and they can spread inside the whole volume of the considered medium. Also, the parameters of the medium become controlled due to the modification of the electromagnetic field.

The importance of the metamaterials in the modern radio engineering and telecommunications is incomparable, and gives possibility to create the absolutely new types of the wave guides, filters, antennae, etc.

The metamaterials have the unique properties in the certain frequency range, in particular, when $\varepsilon, \mu < 0$. The specificity of the metamaterials is determined not so much by the properties of their elements, but by the artificially formed periodic structure.

In reality, those above mentioned objects represent the artificially formed and specially structured media having electromagnetic properties which are not encountered in nature or difficult to be achieved technologically. The latter implies the special characteristics of the physical parameters of the medium. For example, the aforesaid negativity $\varepsilon, \mu < 0$, the spatial structuring (localization) of the distribution of those parameters' values, the possibility in the control of the parameters of the medium, etc. The last fact means the creation of the metamaterials with the electrically controlled dielectric and magnetic permeability.

The prefix "meta" is translated from Greek as "posterior", which makes it possible to interpret the term "metamaterials" in the sense of such structures whose effective electromagnetic features are beyond the properties of their components.

The metamaterials are synthesized by the penetration into the original natural material of various periodic geometric objects. Thus, the dielectric and magnetic permeability of the primary medium are modified.

A special action in the realization of the unique properties for the metamaterials is the design of the microwave devices with the improved wide-band and functional characteristics. It concerns the media combined in pairs with the different signs of μ and ε.

For example, the compounded plates, where $\varepsilon, \mu > 0$, and the metamaterials for which $\varepsilon, \mu < 0$. The specificity of the electrodynamic properties of such paired structures is determined by the electromagnetic field behaviour in the form of the surface waves at the boundary between the interconnected media. The properties of those paired structures are used in construction of the waveguides with the electrically small sizes, forming the basis for the creation of the resonators and the scattering devices whose resonant attributes do not depend on the physical size of the structures.

Hence, taking into account the above mentioned information, it should be noted that the constructive investigation of the electromagnetic wave propagation as in the homogeneous, as in the heterogeneous media where $\varepsilon, \mu > 0; \varepsilon, \mu < 0$ is very important. Moreover, since most of the systems of PDEs with the piecewise constant coefficients remain the basic mathematical models in the current electromagnetism, their analytic research is urgent.

2. THE MAIN ANALYTIC RESULTS

An arbitrary square finite-dimensional system of PDEs over an arbitrary finite-(m)-dimensional numerical space is given

$$\sum_{i=1}^{n} A_{ji} \cdot F_i = f_j; (j = \overline{1,n}). \tag{1}$$

In (1), $\overrightarrow{F}, \overrightarrow{f} = \overrightarrow{F}, \overrightarrow{f}(x_1, \dots, x_m)$ are the n-dimensional unknown and given vector functions respectively. Their corresponding scalar components look like $\{F_i = F_i(x_1, \dots, x_m)\}_{i=1}^{n}$, $\{f_j = f_j(x_1, \dots, x_m)\}_{j=1}^{n}$.

Since the matrix elements A_{ji}, $(j, i = \overline{1,n})$ are the partial differential operators, their commutativity in pairs and invertibility are accepted a priori as opposed to the general case of (1).

At first, the external strategical action of the suggested method is described here. Namely, the present procedure is disjoined into two main stages which are called the "upward" and "downward" diagonalization directions.

The goal of the first stage implies the obtaining of the scalar equation regarding the only one unknown component from the set $\{F_i\}_{i=1}^{n}$. Not breaking the common character of the results, it can be assumed that the sought for function is F_1.

The second stage is opposite to the preceding one, because basing on the obtained first "upper" scalar equation with respect to F_1, this algorithm descends step-by-step to the bottom of the original system. Finally, in consecutive order, all other scalar equations with the components $\{F_i\}_{i=1}^n$ are got.

The tactics of the proposed method deals with the inner action of the diagonalization technique in the limits of both the aforesaid stages, and it is directed at the construction of all scalar equations for the set $\{F_i\}_{i=1}^n$.

2.1. THE "UPWARD" DIAGONALIZATION" STAGE

The given stage begins from

the step 1, where the last equation in (1) is written separately with respect to all other equations, and the summand containing the scalar component F_n is isolated everywhere in (1)

$$\begin{cases} \sum\limits_{i=1}^{n-1} A_{ji} \cdot F_i + A_{jn} \cdot F_n = f_j; (j = \overline{1, n-1}), \\ \sum\limits_{i=1}^{n-1} A_{ni} \cdot F_i + A_{nn} \cdot F_n = f_n. \end{cases} \tag{2}$$

Further, the operators $(-A_{jn}), \forall j = \overline{1, n-1}$, and A_{nn} are applied to the last and other $n-1$ equations in (2) respectively. The consequent sequential summing of the transformed nth equation and the rest $n-1$ equations $\forall j = \overline{1, n-1}$ reduces (2) to the equivalent system

$$\begin{cases} \sum\limits_{i=1}^{n-1} (A_{nn} \cdot A_{ji} - A_{jn} \cdot A_{ni}) \cdot F_i = A_{nn} \cdot f_j - A_{jn} \cdot f_n; \\ (j = \overline{1, n-1}), \\ - - - - - - - - - - - - - - - - - - \\ \sum\limits_{i=1}^{n-1} A_{ni} \cdot F_i + A_{nn} \cdot F_n = f_n. \end{cases} \tag{3}$$

It should be noted, that in the last system and everywhere in the present article, the operator product is understood as the sequential influence, beginning from the inner "factor".

The first $n-1$ equations in (3) have no the scalar component F_n now, and the nth equation which "closes" the appropriate system is called single.

The auxiliary symbols for the known operators and functions

$$A_{nn} \cdot A_{ji} - A_{jn} \cdot A_{ni} = B_{ji}^{(1)}; A_{nn} \cdot f_j - A_{jn} \cdot f_n = g_{j1}; (j, i = \overline{1, n-1}), \tag{4}$$

reduce the first $n-1$ equations from (3) to the following subsystem

$$\sum_{i=1}^{n-1} B_{ji}^{(1)} \cdot F_i = g_{j1}; (j = \overline{1, n-1}) \tag{5}$$

which represents (3) without its single equation.

Simplifying all further calculations, at each next step, the corresponding system will be considered without its single equations. Therefore, the order of each new subsystem becomes less in comparison with all previous.

It should be remembered that at the end of each diagonalization step the single equation is cast out, as it was done for (5). It means the "partial" equivalence between the last and the next subsystems in the framework of each algorithmic step, until all single equations are attached to the final $(n-1)$th result. The latter is "completely" equivalent to the initial systems (1), (2), and this fact will be proved at the end of the present subsection.

Thus, generalizing the above mentioned suggested approach for all inner steps $k = \overline{1, n-1}$ of the "upward" diagonalization procedure, one gets the corresponding system of the step k

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} \cdot F_i = g_{jk}; (j = \overline{1, n-k}; k = \overline{1, n-1}). \tag{6}$$

Subsystem (6) derives the general step $k+1$, $\forall k = \overline{1, n-1}$. Namely, in all $n-k$ equations from (6), the summand with F_{n-k} is isolated as earlier, and the last equation is written separately

$$\begin{cases} \sum\limits_{i=1}^{n-k-1} B_{ji}^{(k)} \cdot F_i + B_{j,n-k}^{(k)} \cdot F_{n-k} = g_{jk}; (j = \overline{1, n-k-1}), \\ \sum\limits_{i=1}^{n-k-1} B_{n-k,i}^{(k)} \cdot F_i + B_{n-k,n-k}^{(k)} \cdot F_{n-k} = g_{n-k,k}; (k = \overline{1, n-1}). \end{cases} \tag{7}$$

The following application of the operator $(-B_{j,n-k}^{(k)})$ to the $(n-k)$th equation from (7) $\forall j = \overline{1, n-k-1}$, and the simultaneous influence of $B_{n-k,n-k}^{(k)}$ to the rest equations of the same system, bring to the next action. In terms of the given "upward" procedure, it implies the further term-by-term consistent addition of the $(n-k)$th and all the rest transformed equations from (7), $\forall j = \overline{1, n-k-1}$. Finally, it gives the equivalent system

$$\left\{\begin{array}{l} \sum\limits_{i=1}^{n-k-1} (B^{(k)}_{n-k,n-k} \cdot B^{(k)}_{ji} - B^{(k)}_{j,n-k} \cdot B^{(k)}_{n-k,i}) \cdot F_i = \\ B^{(k)}_{n-k,n-k} \cdot g_{jk} - B^{(k)}_{j,n-k} \cdot g_{n-k,k}; (j = \overline{1, n-k-1}), \\ -\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,- \\ \sum\limits_{i=1}^{n-k-1} B^{(k)}_{n-k,i} \cdot F_i + B^{(k)}_{n-k,n-k} \cdot F_{n-k} = g_{n-k,k}. \end{array}\right. \tag{8}$$

The first $n-k-1$ equations in (8) have the components $F_i, (i = \overline{1, n-k-1})$ and do not contain $F_i, (i = \overline{n-k, n})$. Here, the last $(n-k)$th equation is single.

The additional auxiliary notations for the known operators and functions

$$\begin{array}{c} B^{(k)}_{n-k,n-k} \cdot B^{(k)}_{ji} - B^{(k)}_{j,n-k} \cdot B^{(k)}_{n-k,i} = B^{(k+1)}_{ji}; \\ B^{(k)}_{n-k,n-k} \cdot g_{jk} - B^{(k)}_{j,n-k} \cdot g_{n-k,k} = g_{j,k+1}; (j, i = \overline{1, n-k}), \end{array} \tag{9}$$

permit rewriting (8) without its single equation

$$\sum_{i=1}^{n-k-1} B^{(k+1)}_{ji} \cdot F_i = g_{j,k+1}; (j = \overline{1, n-k-1}; k = \overline{1, n-1}). \tag{10}$$

In (10), the known operators $B_{\dots}$ and functions $g_{\dots}$ are determined by (4), (9).

The last step $k = n-1$ means the substitution of $k+1 = n-1 \Longleftrightarrow k = n-2$ for (9), (10) and reduction to the desired scalar equation with respect to the component F_1

$$B^{(n-1)}_{11} \cdot F_1 = g_{1,n-1}, \tag{11}$$

while the rest $n-1$ nonscalar equations remain single. In (11), the corresponding known operator and function are given by the following recurrent formulae which are got after substitution of $k = n-2$ for (9):

$$\begin{array}{c} B^{(n-1)}_{11} = B^{(n-2)}_{22} \cdot B^{(n-2)}_{ji} - B^{(n-2)}_{j2} \cdot B^{(n-2)}_{2i}; (j, i = 1), \\ g_{1,n-1} = B^{(n-2)}_{22} \cdot g_{1,n-2} - B^{(n-2)}_{12} \cdot g_{2,n-2}. \end{array} \tag{12}$$

In (9) - (12) and everywhere further, the upper index in the round brackets of the known operator $B_{\dots}$ and the second inferior index of the given function $g_{\dots}$ identify the step number of the "upward" diagonalization stage.

At last, the attachment of all single equations, rejected earlier according to the above mentioned conditions of the present procedure, gives the system which is equivalent to (1)≡(2)

$$\left\{\begin{array}{l} B^{(n-1)}_{11} \cdot F_1 = g_{1,n-1}, \\ \sum\limits_{i=1}^{n-k} B^{(k)}_{ji} \cdot F_i = g_{jk}; (j = \overline{2, n-k}; k = \overline{0, n-2}).(*) \end{array}\right. \tag{13}$$

In (13), $B_{ni}^{(0)} = A_{ni}, (i = \overline{1,n})$; $g_{n0} = f_n$ for $k = 0$, and represent the known operators and function from (3), (4).

The obtained system (13) completes the "upward" diagonalization direction and attains the goal of the present subsection.

2.2. THE "DOWNWARD" DIAGONALIZATION STAGE

At the beginning of the second "downward" diagonalization stage, it should be noted that until the end of this paragraph, the arrow direction for the index k in $(*)$ from (13) describes the opposite indication from the right to the left, e.g. $k = \overleftarrow{0, n-3}$.

Hence, the aforesaid diagonalization stage begins, and its first

step 1 $(k = n-2)$ concerns the isolation of the first equation in the subsystem $(*)$ from (13), simultaneously considering this equation together with the first scalar one from (13). Here, the rest $k = \overleftarrow{0, n-3}$ equations from $(*)$ in (13) are accepted as single:

$$\begin{cases} B_{11}^{(n-1)} \cdot F_1 = g_{1,n-1}; \\ \sum\limits_{i=1}^{2} B_{ji}^{(n-2)} \cdot F_i = g_{j,n-2}; (j = 2). \end{cases} \tag{14}$$

Adhering to the general algorithm of the considered method, the summand with the scalar F_2 is isolated in the last equation from (14). The influence of the operators $B_{11}^{(n-1)}$, $(-B_{21}^{(n-2)})$ to the second and the first equations in (14), together with the further summing of those both transformed equations gives the equivalent system

$$\begin{cases} B_{11}^{(n-1)} \cdot F_1 = g_{1,n-1}; \\ B_{11}^{(n-1)} \cdot B_{22}^{(n-2)} \cdot F_2 = h_1. \end{cases} \tag{15}$$

In (15), the second scalar equation regarding F_2 appears, and

$$h_1 = B_{11}^{(n-1)} \cdot g_{2,n-2} - B_{21}^{(n-2)} \cdot g_{1,n-1} \tag{16}$$

is the known function.

It is clear that after obtaining (15), the subsystem $(*)$ from (13) diminishes by one equation and looks like

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} \cdot F_i = g_{jk}; (j = \overline{3, n-k}; k = \overleftarrow{0, n-3}), \tag{17}$$

where the index of the known function $h_{\dots}$ from (16) and everywhere further in this subsection, means the step number of the second "downward" diagonalization stage.

The generalization of the current "downward" diagonalization direction in the case of an arbitrary step l, $(l = \overline{1, n-1})$, first of all brings to the following subsystem of the preceding step $l-1$

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} \cdot F_i = g_{jk}; (j = \overline{l+1, n-k}; k = \overleftarrow{0, n-l-1}). \tag{18}$$

For $l = 1$, the second equation in (14) corresponds to the "zero" step of the present diagonalization stage.

According to the general approach of the proposed here diagonalization procedure, the first equation from (18) is isolated and attached to the final system of the scalar equations in the previous step $l-1$. At the same time, all other $k = \overleftarrow{0, n-l-2}$ equations in (18) are single.

Hence, the required system whose last equation is scalar, looks like

$$\begin{cases} (\prod_{q=1}^{p+1} B_{qq}^{(n-q)}) \cdot F_{p+1} = h_p; (p = \overline{0, l-1}; h_0 = g_{1,n-1}), \\ \sum_{i=1}^{l+1} (B_{ji}^{(n-l-1)} \cdot F_i) = g_{j,n-l-1}; (j = l+1). \end{cases} \tag{19}$$

In (19) and everywhere in the present paragraph, the symbol of the finite operator "product" means the sequential operator action in the right to the left direction, from the inner to the outer side.

Further, according to the diagonalization scheme, the summand with the component F_{l+1} is isolated in the $(l+1)$th equation from (19). Then, the operators $\prod_{q=1}^{l} B_{qq}^{(n-q)}$ and $(-B_{l+1,r}^{n-l-1} \cdot \prod_{q=r+1}^{l} B_{qq}^{(n-q)})$, $(r = \overline{1, l-1})$ are applied to the last equation in (19) and to the remainder respectively. Transformation of the lth equation from (19) using operator $(-B_{l+1,l}^{(n-l-1)})$, after summing of all other $(l+1)$ equations, gives the equivalent system

$$\begin{cases} (\prod_{q=1}^{p+1} B_{qq}^{(n-q)}) \cdot F_{p+1} = h_p; (p = \overline{0, l-1}; h_0 = g_{1,n-1}), \\ (\prod_{q=1}^{l+1} B_{qq}^{(n-q)}) \cdot F_{l+1} = (\prod_{q=1}^{l} B_{qq}^{(n-q)}) \cdot g_{l+1,n-l-1} - \\ \sum_{r=1,(l \neq 1)}^{l-1} B_{l+1,r}^{(n-l-1)} \cdot (\prod_{q=r+1}^{l} B_{qq}^{(n-q)} \cdot h_{r-1}) - B_{l+1,l}^{(n-l-1)} \cdot h_{l-1}. \end{cases} \tag{20}$$

In the case of $l = 1$, the second summand at the right part of the last equation in (20) is assumed to be equal to zero.

The suggested below notation for the known function from the right part of the last equation in (20)

$$\begin{gathered} h_l = (\prod_{q=1}^{l} B_{qq}^{(n-q)}) \cdot g_{l+1,n-l-1} - \\ \sum_{r=1,(l\neq 1)}^{l-1} B_{l+1,r}^{(n-l-1)} \cdot (\prod_{q=r+1}^{l} B_{qq}^{(n-q)} \cdot h_{r-1}) - B_{l+1,l}^{(n-l-1)} \cdot h_{l-1} \end{gathered} \tag{21}$$

allows rewriting the final system of an arbitrary step $l, (l = \overline{1, n-2})$ in the following manner

$$(\prod_{q=1}^{p+1} B_{qq}^{(n-q)}) \cdot F_{p+1} = h_p; (p = \overline{0, l}; h_0 = g_{1,n-1}). \tag{22}$$

In (22), the second summand at the right part of (21) equals zero while $l = 1$.

The obtained recurrent formulae (21), (22) are easily checked, for example, while $l = 1, 2$.

It should be noted that after the formation of (22), the subsystem (18) decreases by one equation and looks like

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} \cdot F_i = g_{jk}; (j = \overline{l+2, n-k}; k = \overleftarrow{0, n-l-2}). \tag{23}$$

Simultaneously, the original system $(*)$ from (13) decreases by l equations correspondingly.

The continuation of the "downward" diagonalization stage, until the step $l = n-1$, leads to the required system of the scalar equations with respect to all components $F_i, (i = \overline{1, n})$

$$(\prod_{q=1}^{p+1} B_{qq}^{(n-q)}) \cdot F_{p+1} = h_p; (p = \overline{0, n-1}; h_0 = g_{1,n-1}). \tag{24}$$

In (24), the known operators and functions are described in (9), (21), and (24) becomes (22) if $l = n-1$.

After ending the concluding system (24) construction, one can assert the nonexistence of the subsystem consisting of the single equations (23). It is true, since after the completion of the previous step $l = n-2$, the subsystem (23) contains only one equation

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} \cdot F_i = g_{jk}; (j = \overline{n, n-k}; k = 0) \Longleftrightarrow \sum_{i=1}^{n} B_{ni}^{(0)} \cdot F_i = g_{n0}. \qquad (25)$$

In (25), the known operators $B_{...}$ and function $g_{...}$ are given by the expressions which are written exactly below the system (13), and the described above final step $l = n-1$ reduces (25) to the required scalar equation with the component F_n.

2.3. THE PRINCIPAL RESULT

It should be noted, that the wanted system of the scalar equations (22) is equivalent to the final system (13) of the "upward" diagonalization procedure, and both of them are equivalent to the initial system (1) from the original problem statement. This assertion is the direct corollary of the proposed diagonalization procedure and completes it.

The final result can be expressed as the following

Theorem 2.1. *The constructive solution of the general differential system of PDEs* (1) *using the operator diagonalization method exists and is obtained as an exact algorithm. The present technique represents the substantial generalization of the algebraic Gauss procedure.*

The proof of the theorem is performed at the subsections 2.1, 2.2, relying on the realization of all steps in the suggested diagonalization method.

3. MATHEMATICAL PRELIMINARIES REGARDING THE HETEROGENEOUS MEDIUM

Considering the information from Section 1 concerning the heterogeneous media and taking into account the corresponding engineering claims / restrictions described there, the relevant formulae for σ, ε, μ are suggested below.

At first, it is reasonable to consider the classical case of the differential Maxwell statement in the Cartesian coordinate system (x, y, z, t), where the vector field function looks like

$$\vec{F}(x, y, z, t) = \sum_{i=1}^{3} F_i(x, y, z, t) \cdot \vec{e}_i. \qquad (26)$$

In (26), vectors

$$\vec{e}_1 = \vec{i}, \vec{e}_2 = \vec{j}, \vec{e}_3 = \vec{k} \qquad (27)$$

form the ordinary orthonormal basis, and $F_i(x,y,z,t),(i=\overline{1,3})$ are the respective scalar components (scalar fields) of $\vec{F}(x,y,z,t)$.

Hence, in terms of (26), (27), the field parameters under the earlier mentioned electromagnetic requirements can be expressed as follows

$$\begin{bmatrix}\sigma\\ \varepsilon\\ \mu\end{bmatrix} = \begin{bmatrix}\sigma\\ \varepsilon\\ \mu\end{bmatrix}(x,y,z,t) = \sum_{l=1}^{s}\left(\begin{bmatrix}{}_l\sigma\\ {}_l\varepsilon\\ {}_l\mu\end{bmatrix}\cdot\delta(x,y,z;{}_lV)\right), \forall s\in\mathbb{N}. \tag{28}$$

In (28): $\delta(x,y,z;{}_lV) = \begin{cases} 1,(x,y,z)\in {}_lV \\ 0,(x,y,z)\in V-{}_lV, \end{cases}$, $(l=\overline{1,s})$ is the Kronecker symbol; $V=\bigcup\limits_{l=1}^{s} {}_lV$ is the finite union of the various three-dimensional spatial media ${}_lV={}_lV(x,y,z)$, where ${}_lV\bigcap {}_\nu V=\emptyset$ for $\nu\neq l,(l,\nu=\overline{1,s})$, and each ${}_lV,(l=\overline{1,s})$ has its own specific field features accepted as ${}_l\sigma, {}_l\varepsilon, {}_l\mu=$ **const**, $(l=\overline{1,s})$.

Returning again to Section 1, it should be reminded about the numerical values of ${}_l\varepsilon, {}_l\mu$, $(l=\overline{1,s})$ which can be also negative. And in this case, the coincidence of their signs is the obligatory condition for the electromagnetic wave propagation [10] - [13].

Further, it is obvious that all partial differential operators in the classical Maxwell system for the space (x,y,z,t) [9] are formed by the superpositions of

$$\partial_k=\frac{\partial}{\partial x_k},(k=\overline{0,3});x_0=t,x_1=x,x_2=y,x_3=z. \tag{29}$$

Therefore,

$$\begin{gathered}\partial_k\left(\begin{bmatrix}\sigma\\ \varepsilon\\ \mu\end{bmatrix}\cdot F_i(x,y,z,t)\right)= \\ \partial_k\left(\left(\sum_{l=1}^{s}\begin{bmatrix}{}_l\sigma\\ {}_l\varepsilon\\ {}_l\mu\end{bmatrix}\cdot\delta(x,y,z;{}_lV)\right)\cdot F_i(x,y,z,t)\right)= \\ \left(\sum_{l=1}^{s}\begin{bmatrix}{}_l\sigma\\ {}_l\varepsilon\\ {}_l\mu\end{bmatrix}\cdot\delta(x,y,z;{}_lV)\right)\cdot\partial_kF_i(x,y,z,t),(i=\overline{1,3};k=\overline{0,3}).\end{gathered} \tag{30}$$

Using (30) and (26), the following formula is obtained

$$\partial_k \left(\begin{bmatrix} \sigma \\ \varepsilon \\ \mu \end{bmatrix} \cdot \vec{F}(x,y,z,t) \right) =$$
$$\left(\sum_{l=1}^{s} \begin{bmatrix} {}_l\sigma \\ {}_l\varepsilon \\ {}_l\mu \end{bmatrix} \cdot \delta(x,y,z;{}_lV) \right) \cdot \partial_k \left(\sum_{i=1}^{3} F_i(x,y,z,t) \cdot \vec{e}_i \right) = \tag{31}$$
$$\left(\sum_{l=1}^{s} \begin{bmatrix} {}_l\sigma \\ {}_l\varepsilon \\ {}_l\mu \end{bmatrix} \cdot \delta(x,y,z;{}_lV) \right) \cdot \left(\sum_{i=1}^{3} \partial_k F_i(x,y,z,t) \cdot \vec{e}_i \right), (k = \overline{0,3}).$$

In (30), (31), the linearity of partial derivatives (29) and formulae (26) - (28) are used.

Expressions (30), (31) reduce the original matrix statement with the piecewise constant coefficients to the finite totality of $s \in \mathbb{N}$ systems of PDEs with arbitrary constant coefficients ${}_l\sigma, {}_l\varepsilon, {}_l\mu$, $(l = \overline{1,s})$, where $\mathbf{sgn}\ {}_l\varepsilon = \mathbf{sgn}\ {}_l\mu$, $(l = \overline{1,s})$.

4. CONCLUSIONS

The advantages of the proposed here analytic technique are the following:

1. In general, in the present operator diagonalization procedure, all matrix elements should be invertible and commutative in pairs. For the systems of PDEs such requirements are fulfilled automatically.

2. Both preceding conditions support the a'priori existence of the inverse operator elements, without their direct explicit construction.

3. The proposed apparatus allows applying the given procedure to the systems of PDEs with the piecewise constant coefficients. Such mathematical objects are the important models of the electrodynamic engineering processes in the heterogeneous media.

4. This general differential operator diagonalization technique gives the unified formulae for all scalar equations regarding the components of the originally unknown vector field function.

5. The proposed method remains valid for the matrices with the various block structure and does not depend on the boundary and initial conditions which are formulated later, when the scalar equations are got.

6. The mathematical simulation of the physical / engineering processes using the relevant boundary value problems is easier to do in terms of the scalar equations than for the original matrix statement. In general, the analysis and constructive solution of the latter are complicated and sometimes even unobservable.

Concerning the shortcomings of the present research, it should be noted that:

1. Though the theoretical and numerical results regarding the spatial electrodynamic process mentioned in the abstract are finished, they are not proposed just in this paper because of the lack of space. The analytic solution, numerical implementation and computer modelling are planned for the next article, Part 2.

2. The suggested explicit results for the heterogeneous media are not applied yet to the specific electromagnetic phenomenon. The problem is solved only partially, and its complete study is expected in the nearest future.

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