

LUMPED PARAMETER MODEL FOR STEADY STATE HEAT TRANSFER CALCULATION IN INTERCONNECTED ELECTRICAL SYSTEMS AND FLUIDS

Oliver Magdun*, Alin Pohoată**, Otilia Nedelcu, Corneliu Sălișteanu

Valahia University of Târgoviște, Târgoviște, Romania

oliver.magdun@valahia.ro, alin.pohoata@valahia.ro, otilia.nedelcu@valahia.ro, cornel.salisteanu@valahia.ro

*main author, **corresponding author

Abstract Applying lumped thermal model networks to interconnected electrical systems and fluids, in analogy with the electrical circuit methods, the system temperatures can be advantageously calculated in every node of the thermal networks with a complete system of equations, which considers the geometry, heating sources, fluid properties and material characteristics. In this paper, the lumped model parametrization for steady-state calculation is explained, and comparisons of the calculated results with the lumped model and with the computational fluid dynamic tools are presented. For example, the heat transfer is calculated here in the case of a) – heating of cooled hollow conductors and b) – heat exchangers of a water heating system.

Keywords: lumped parameter, thermal model, heat transfer coefficients, computational fluid dynamic.

2010 MSC: 80A05; 80A20.

1. INTRODUCTION

With the advance and implementation of the newest technologies in the household and industrial applications, the forced cooling is regularly used in order to prevent the overheating and oversizing of almost every efficient equipment. Reducing the size, weight and price is possible by optimal cooling, which finally results in a further increase of the utilization coefficient. The optimal cooling is achieved when the accurate calculation of the system temperatures is performed in the design stage. Generally, for calculating the heat transfer from the electrical systems to the heat transportation medium, lumped thermal models [1], [2], coupled electro-thermal finite element (FE) methods [3], [4] or high-performance computational fluid dynamic (CFD) tools [5] are used. In order to use lumped thermal models or electro-thermal FE models, heat transfer coefficients are required [6]. If the heat transfer coefficients are well determined and the lumped model networks are identified based on verified models, i.e. for particularly cases, the calculation results of the thermal

models fit quite well with the measurements [6]. In many cases the heat transfer coefficients are not known and they have to be calculated, as an example, following the methods of [7] or using the CFD tools. In this paper, both methods are used for calculating the heat transfer coefficients. Further, the heat transfer from the heated components to the cooling fluids is calculated based on a lumped thermal model. In order to generate a thermal network model, an “in-house” software, able to assembly the system of equations for different geometries and physical interconnections, was written. For that, the interconnected solid-fluid structure is subdivided in small elements and the energy balance equation and Kirchhoff’s circuit laws are applied for each element of the considered structure. The parameters of the equation system (losses, thermal resistances, fluids) depend on the temperature, so the system of equation needs to be solved repeatedly, following an iterative resolution process. For verifying the lumped parameter model results and the software as well, two examples have been chosen and presented: the heat transfer calculation for a) - a heated hollow conductor and b) - a heat exchange. The lumped thermal model calculation method with consideration of the energy balance equation can be extended for several complicate structures, i.e. electrical machines, power converters or battery cooling systems, some of these examples are to be presented in a further paper.

2. HEAT TRANSFER

In the following, due to particular applications of the thermal models of this paper, only the heat transfer by conduction and convection are considered, whereas the heat transfer by radiation is neglected.

2.1. HEAT TRANSFER BY CONDUCTION

Transfer of heat from a part of a body with a high temperature ϑ_1 to another part of the same body with a low temperature ϑ_2 is done by conduction. According to the Fourier’s law, in steady-state heat transfer problems, the heat flux caused by a temperature difference in the material $\Delta = \vartheta_1 - \vartheta_2$ is proportional to the spacial temperature gradient and the thermal conductivity of the material λ :

$$\vec{q} = -\lambda \nabla \vartheta \quad (1)$$

. By assuming the heat transfer only in the Ox direction of a cartesian coordinate system, the heat flux density is given by:

$$q = -\lambda \frac{d\vartheta}{dx}, \quad (2)$$

Equation (2) can be integrated:

$$\int_{x_1}^{x_2} q dx = - \int_{\vartheta_1}^{\vartheta_2} \lambda d\vartheta \quad (3)$$

$$q = \frac{\lambda}{x_2 - x_1} (\vartheta_1 - \vartheta_2), \quad (4)$$

Thus:

$$q = \frac{1}{l} \lambda \Delta\vartheta, \quad (5)$$

where $l = x_2 - x_1$ is a defined length along the heat transfer in the body and x_1 and x_2 are the coordinates of the locations where the material temperatures are ϑ_1 and ϑ_2 . By analogy with the Ohm's law, a thermal resistance due to conduction, related to a body with homogeneous properties, can be defined:

$$R_{th,cond} = \frac{\Delta\vartheta}{qA} = \frac{l}{\lambda A}, \quad (6)$$

where: qA is the heat flux crossing a body with the regular surface A along the heat transfer direction of length l .

2.2. HEAT TRANSFER BY CONVECTION

Transfer of heat from a bounding surface to a moving fluid is done by convection. For the heat transfer calculation, convection heat transfer coefficients are used, which are described as functions of the flow conditions, physical properties of the medium, geometry and temperature distribution. A comprehensive study for the calculation of the convection heat transfer coefficients is presented in [7], here the thermal network resistance model is proved for only two applications, both considering cylindrical tubes crossed by fluids as heat transportation mediums. Hence, formulas for calculating the heat transfer coefficients are presented here only for the cases of (Fig.3). By using the hydraulic diameter instead of the inner cylinder diameter, hollow conductors of inner different shapes may be analysed similarly [8].

A dimensionless heat transfer coefficient is the *Nusselt* number Nu . It can be analytically determined for laminar flow, whereas for turbulent flow it is just an empirical result of the experimental tests, fitting with the theory approach. Determination of the heat transfer coefficients is therefore reduced to determination of the Nu for every particular situation. With the *Nusselt* numbers, the convection heat transfer coefficients can be calculated:

$$\alpha_{\vartheta} = Nu \frac{\lambda}{l} \quad (7)$$

where λ is the thermal conductivity of the fluid and l is the length of the tube in the flow direction. A thermal resistance, caused by the heat transfer by convection, is defined according to the Newton's law:

$$R_{th,conv} = \frac{\Delta\vartheta}{qA} = \frac{1}{\alpha_{\vartheta}A} \quad (8)$$

which is further used in the system of equations describing the lumped thermal model (Section 3).

a) Laminar flow in cylindrical tubes

In very long tubes, at laminar flow, the *Nusselt* number is $Nu_{lam} = 3.66$ [7]. For shorter tubes with the inner diameter d_i and the length l , the *Nusselt* number is given by:

$$Nu_{lam} = 0.644 \sqrt[3]{Pr} \sqrt{Re \frac{d_i}{l}} \quad (9)$$

where the influence of the material properties is considered by the *Prandtl* number: $Pr = \eta \frac{c_p}{\lambda}$, with η - the dynamic viscosity, c_p - the specific heat capacity at constant pressure and λ - the thermal conductivity of the fluid, and the flow parameters are considered by the *Reynold* number: $Re = \frac{\nu l \rho}{\eta}$, with ν - the average flow velocity and ρ - the mass density of the fluid.

For Reynold numbers below 2300, the *Nusselt* number is given by:

$$Nu_{lam} = \sqrt[3]{3.66^3 + 0.64^3 Pr \left(Re \frac{d_i}{l} \right)^{3/2}} \quad (10)$$

b) Turbulent flow in cylindrical tubes

According to [7], for turbulent fluid flow in cylindrical tubes, assuming the temperature of the fluid in the centre of the tube, the *Nusselt* number is given with a validity range of $10^4 < Re < 10^6$ and $\frac{l}{d_i} < 1$:

$$Nu_{turb} = \frac{(\xi/8) Re Pr}{1 + 12.7 \sqrt{\xi/8} (Pr^{2/3} - 1)} \left[1 + \left(\frac{d_i}{l} \right)^{2/3} \right] f_k \quad (11)$$

where the friction hydraulic factor is:

$$\xi = [1.8 \log(Re) - 1.5]^{-2} \quad (12)$$

The temperature of the fluid in the center of the tube ϑ , which is used for the calculation of the *Prandtl* number and *Reynold* number, is different from the temperature at the tube wall ϑ_w . Thus, corrections of the *Nusselt* number are considered by a factor f_k [8] given by:

$$f_k = \begin{cases} \left(\frac{Pr}{Pr_w} \right)^{0.11} & , \text{ for liquids} \\ \left(\frac{\vartheta}{\vartheta_w} \right)^{0.45} & , \text{ for gases} \end{cases} \quad (13)$$

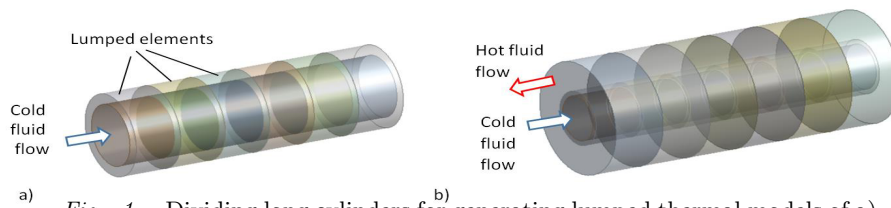


Fig. 1. Dividing long cylinders for generating lumped thermal models of a) - heated hollow conductors and b) - heat exchangers

Additional correlations of the *Nusselt* number, depending on the heat flux directions are indicated in [7]. In this paper *Nusselt* number is a constant calculated for each convection surface and the convection heat transfer coefficient as well.

3. LUMPED MODELS FOR HEAT TRANSFER CALCULATIONS

A solid body, as an example, a heated hollow conductor, crossed through by a coolant (demineralized water), with an inner diameter d_i and an outer diameter d_o , may be divided along its length (Fig.3) in a finite number of homogeneous elements: lumped elements [9], each element having interconnection to the neighboring elements through a linear mesh of thermal resistances. Following the heat transfer direction, from the hollow conductor to the coolant, a network of thermal resistances can be built (Fig.2). Boundary nodes at the subdivided elements defines the sections with constant temperatures of the hollow conductor. The nodes of the thermal network model can be fixed in the centre of the subdivided elements as in the conventional models. The model is simplified under assumption of an ideal insulation of the conductor at the outer surface, such us the heat will not be transferred to the external ambient, but only to the inner coolant by conduction and convection. Therefore, the produced heat is transported by the coolant from the inlet to the outlet of the hollow conductor.

The studies of [9] show that in some cases, depending on the load conditions, the conventional models with centric nodes may fail in finding the correct results. A solution has been proposed in [9], but the nodes have to be shifted from the centre of the structures and several algorithms have to be applied for adjusting the load conditions. Thus, the centric thermal model used in this

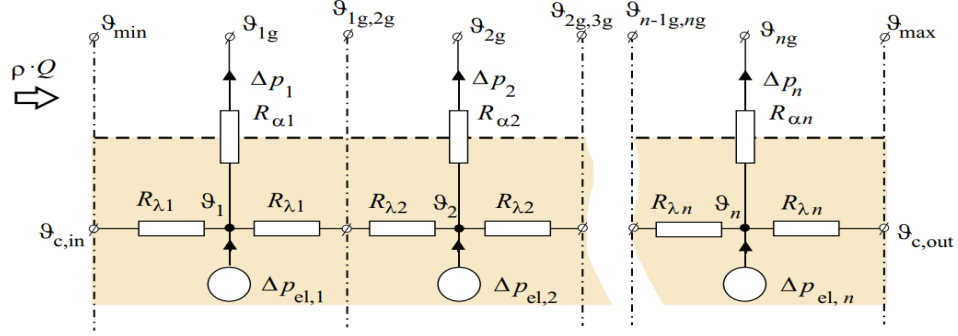


Fig. 2. Thermal network model with lumped elements

paper is verified for the studied particular applications (Fig.3) with ANSYS CFD tools [10].

The internal heat sources of the interconnected electrical systems and fluids, particularly for this case, are the electrical losses:

$$P_{el} = R_{\vartheta} [1 + \alpha (\vartheta_{Cu} - \vartheta)] I^2, \quad (14)$$

where I is the electric current in the hollow conductor (DC or AC rms value), R_{ϑ} is the electrical resistance of the conductor at the temperature of reference ϑ , including skin and proximity effect corrections for AC calculations, α is the coefficient of the resistance increase with the temperature and ϑ_{Cu} is the copper hollow conductor temperature. Additional losses are the friction losses at the contact surface between the solids and fluids:

$$P_{fr} = \Delta p \cdot Q \quad (15)$$

where Δp is the pressure drop and Q is the flow rate of the fluid crossing the tube. At low fluid velocities or very small pressure differences these friction losses are negligible. If the pressure drop is known then for any k element we have:

$$P_{fr,k} = \frac{\Delta p Q}{n} \quad (16)$$

For the circuit of (Fig.2), the energy conservation law results in a general equation, which depends on the total losses $\sum P$, the coolant properties c_p , ρ ,

the flow rate Q and the temperatures of the coolant at the inlet ϑ_{min} and the outlet ϑ_{max} :

$$\sum P = c_p \rho Q (\vartheta_{max} - \vartheta_{min}) \quad (17)$$

and specifically, for the circuit of (Fig.2) with n lumped elements, due to superposition, we get:

$$\begin{aligned} \Delta p_{fr,1} &= 2c_p \rho Q (\vartheta_{1g} - \vartheta_{min}) - (\vartheta_1 - \vartheta_{1g}) G_{\alpha 1} \\ \Delta p_{fr,2} &= c_p \rho Q (\vartheta_{2g} - \vartheta_{1g}) - (\vartheta_2 - \vartheta_{2g}) G_{\alpha 2} \\ &\vdots \\ \Delta p_{fr,n} &= c_p \rho Q (\vartheta_{ng} - \vartheta_{n-1g}) - (\vartheta_n - \vartheta_{ng}) G_{\alpha n} \end{aligned} \quad (18)$$

Applying the *Kirchhoff's* current law for the network of (Fig.2), the connections equations are obtained:

$$\begin{aligned} \Delta p_{el,1} + (\vartheta_2 - \vartheta_1) G_{\lambda 1,2} &= (\vartheta_1 - \vartheta_{1g}) G_{\alpha 1} + (\vartheta_1 - \vartheta_{c,in}) G_{\lambda 1} \\ \Delta p_{el,2} + (\vartheta_3 - \vartheta_2) G_{\lambda 2,3} &= (\vartheta_2 - \vartheta_{2g}) G_{\alpha 2} + (\vartheta_2 - \vartheta_1) G_{\lambda 2,3} \\ &\vdots \\ \Delta p_{el,n} + (\vartheta_{c,out} - \vartheta_n) G_{\lambda n} &= (\vartheta_n - \vartheta_{ng}) G_{\alpha n} + (\vartheta_n - \vartheta_{n-1}) G_{\lambda n-1,n} \end{aligned} \quad (19)$$

where the model conductances: $G_{\alpha k} = \frac{1}{R_{\alpha k}}$, $G_{\lambda k-1,k} = \frac{1}{R_{\lambda k-1,k}} = \frac{1}{R_{\lambda k-1} + R_{\lambda k}}$, $G_{\lambda k} = \frac{1}{R_{\lambda k}}$ are defined for the lumped elements of the circuit model $k = 1 \dots n$. On the direction of the heat transfer, from the conductor to the coolant, the thermal resistance due to conductivity may be neglected for the analyzed cases of (Fig.3). The system of equations (18) and (19) forms a complete equation system with $2n$ equations and $2n$ unknowns per lumped element ϑ_k and ϑ_{kg} . The temperature of the coolant at the inlet ϑ_{min} is known. Further, for a large number of lumped elements, we can impose: $\vartheta_{max} \cong \vartheta_{ng}$, $\vartheta_{c,in} \cong \vartheta_1$ and $\vartheta_{c,out} \cong \vartheta_n$. The electric losses are uniformly distributed, thus for any k element input: $\Delta p_{el,k} = P_{el}/n$. However, depending on the application, the loss distribution can be particularly arranged. For more details on the mathematical formulation of the model we recommend [11] and [12].

4. COOLED HOLLOW CONDUCTORS

A hollow copper conductor of 3 m length with the inner diameter $d_i = 20$ mm and the outer diameter $d_o = 45$ mm is heated up by electrical losses $P_{loss} = 12.5$ kW, as an example for the terminal connections of a large power synchronous generator. The cooling is made with demineralized water with very low conductivity. At the inlet, the coolant temperature is 40 °C and the velocity is 1 m/s. The system temperatures are obtained by solving the

equation systems (18) and (19), which are coupled by an “in-house” program, which can assemble the conductance matrix, according to the imposed geometry and material properties. The hollow conductor and the fluid are divided in 50 elements, hence a system of 100 equations results. The calculated temperatures of the hollow conductor (continuous red line) and of the coolant (dotted red line) are given in (Fig.3).

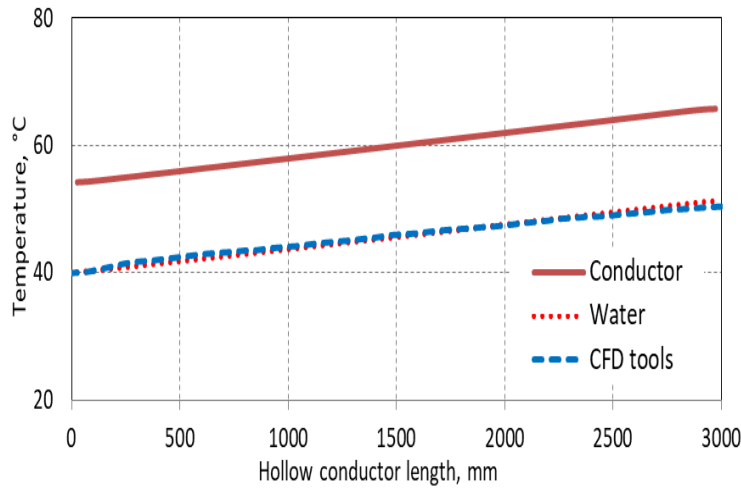


Fig. 3. Calculated temperatures of the coolant of the hollow conductor with the method presented in the paper and with the ANSYS CFD tools

It is supposed that no heat transfer to the ambient occurs via the outer surface of the hollow conductor. For solving this problem with the ANSYS CFD, only the inner part of the hollow conductor is considered and this part is discretized in finite elements. For a better consideration of the border layers which are formed by the fluid flow at the contact surface with the copper material of the hollow conductor, inflation layers are introduced in the mesh (Fig.4). Boundary conditions are applied at the inlet, e.g. the temperature and fluid velocity, and at the inner surface of the hollow conductor where the heat caused by the electrical losses flows from the conductor to the coolant. When CFD tools are used for solving interconnected electrical systems and fluids, not only the temperatures of the coolant and conductor can be determined (Fig.5), but also the fluid velocity (Fig.6) and heat transfer coefficient (Fig.7) are provided.

An additional verification of the method presented in this paper is performed in the case of a heat exchanger.

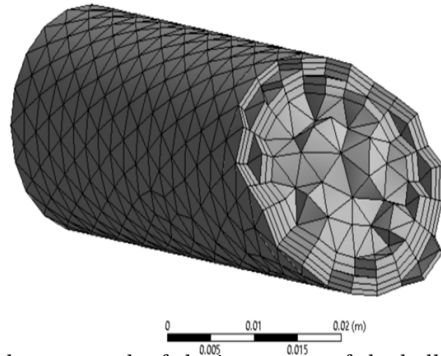


Fig. 4. Finite element mesh of the inner part of the hollow conductor with inflation layers

5. HEAT EXCHANGERS

A heat exchanger is built from an outer tube with the length of 3.63 m and the outer diameter of 24 mm and a central inner tube with the outer diameter of 18 mm and the wall thickness of 1 mm [8]. The service water flows with a velocity of 1 m/s and it has a temperature of 40 °C at the cold-water inlet.

	ρ [kg/m ³]	η/ρ 10 ⁻⁶ [m ² /s]	λ [W/(mK)]	c_p [J/Kg]	Pr
Service water, 40 °C, inlet	998.1	0.553	0.644	4179	3.55
Heating water, 90 °C, outlet	971.8	0.365	0.670	4195	2.22

Table 1 Properties of the service water and of the heating water

The heating water counter-flows with the velocity 1m/s and it has a temperature of 90 °C at the hot-water inlet. It is assumed that no heat is transferred

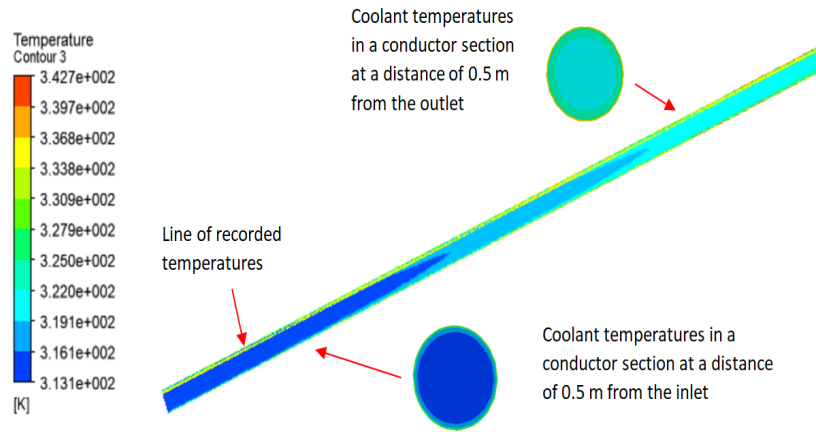


Fig. 5. Representation of the coolant temperature along the hollow conductor length. The flows effect of the fluid on the heat transfer is visible and it is considered in the presented method of the paper.

to the ambient via the outer tube surface. The thermal conductivity of the tubes material is 17 W/(mK) . In Table 1, the properties of the service and heating water are given. According to the calculation of [8], the service water temperature increases from 40°C at the inlet to 60°C at the outlet while the heating water temperature from 90°C at the inlet to 69.2°C at the outlet. The water flow is turbulent. With the proposed algorithm of this paper, the obtained results fit perfectly with the results of [8] and with the results obtained with the ANSYS CFD tools (Fig.8). The water temperature distribution in the longitudinal section of the heat exchanger, calculated with the ANSYS CFD is presented in (Fig.9).

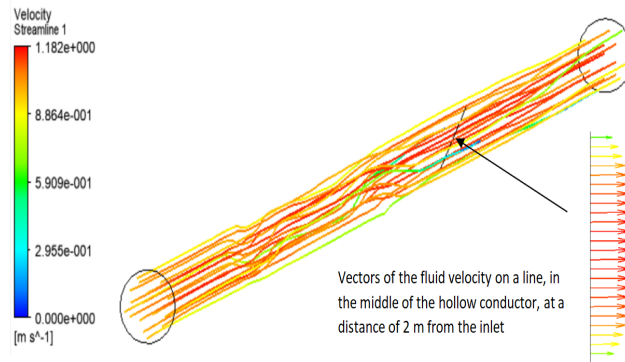


Fig. 6. Streamlines of the coolant flow in the hollow conductor. The maximum calculated velocity of the fluid is 1.18 m/s, turbulences occurring along the hollow conductor. At the contact surface between conductor and fluid, the fluid velocity decreases at nearly half of the maximum velocity.

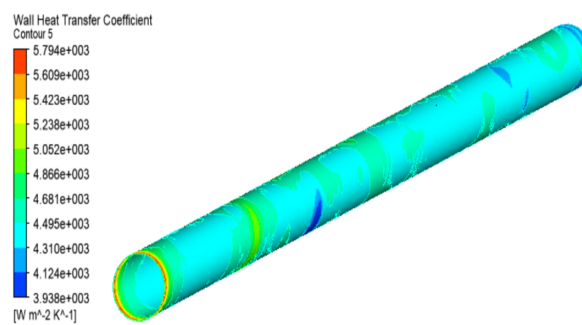


Fig. 7. Heat transfer coefficient at the inner surface of the hollow copper conductor, which includes the turbulence effect of the fluid flow

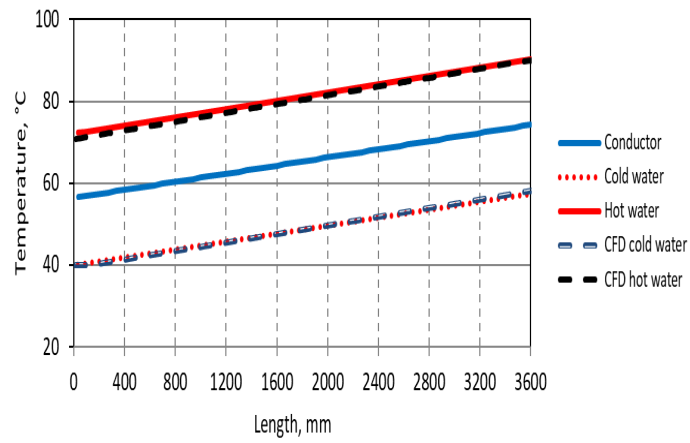


Fig. 8. Calculated temperatures of the service water and heating water of the heat exchanger with the method presented in the paper (continuous and dotted lines) and with the ANSYS CFD tools (dashed lines)

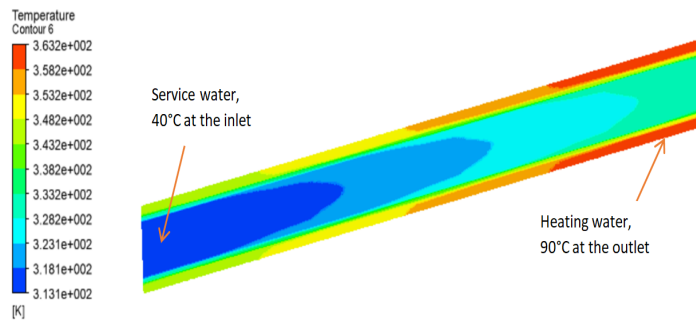


Fig. 9. Water and inner tube temperatures along the heat exchanger

6. CONCLUSIONS

Two applications, which involve the thermal calculation of forced cooled hollow conductor heated by the electrical lanes and a heat exchanger, are used for verifying a lumped thermal network model theory and the algorithm of assembling iterative systems of equations obtained from the energy balance law and the Kirchhoff's circuit laws. For proper results, in the interconnected electrical systems and fluids the energy balance law should not be neglected when thermal network models are used. The good fitting between the calculated results with the presented method and the numerical results of the ANSYS CFD tools shows a very promising alternative for calculating complicate geometries of interconnected electrical systems and cooling configurations with thermal network models. However, in order to certify and to extend the method, a further study should investigate applications with more complicate configurations and it should additionally consider the transient effects.

References

- [1] L. Codecasa, D. D'Amore, P. Maffezzoni, *Compact modeling of electrical devices for electrothermal analysis*, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, **50**, 4(2003), 456-476.
- [2] D. Kuehbach, A. Kelleter, D. Gerling, *An Improved Approach for Transient Thermal Modeling using Lumped Parameter Networks*, Proceedings of IEEE, IEMDC 2013, Chicago, USA, May 12-15, 2013.
- [3] X. Xu, P. Lyu, S. Yan, Z. Ren, *Stationary electro-thermal coupling analysis considering dual finite element formulations of steady current field*, Proceedings of IEEE, CEFC 2016, Miami, USA, November 13 - 16, 2016.
- [4] M. A. Bella, C. Bailey, H. Lu, *Electro-thermal behaviour using finite volume and Finite Element method*, Proceedings of IEEE EuroSimE 2018, Toulouse, France, April 15-18, 2018.
- [5] Z. Sun , J. W. Chew , N. J. Hills , K. N. Volkov , C. J. Barnes, *Efficient Finite Element Analysis/Computational Fluid Dynamics Thermal Coupling for Engineering Applications*, ASME Journal of Turbomachinery, **132**, 3(July 2010).
- [6] D. A. Staton, A. Cavagnino, *Convection Heat Transfer and Flow Calculations Suitable for Electric Machines Thermal Models*, IEEE Transactions on Industrial Electronics, **55**, 10(2008), pp.3509-3516.
- [7] *VDI-Wärmeatlas* (2013), Springer - Verlag, Berlin, Heidelberg, 2013.
- [8] P. von Böckh, T. Wetzel, *Heat transfer. Basics and Practice*, Springer-Verlag, Berlin, Heidelberg, 2012.
- [9] D. Gerling, G. Dajaku, *Novel lumped-parameter thermal model for electrical systems*, Proceedings of IEEE EPE 2005, CD-ROM, Dresden, Germany, September 11-14, 2005.
- [10] *ANSYS Fluent User's guide*, available online at <http://www.ansys.com>
- [11] J.H. Lienhard IV, J.H. Lienhard V, *A Heat Transfer Textbook*, Phlogiston Press, Cambridge 2008

100 *Oliver Magdun, Alin Pohoăță, Otilia Nedelcu, Corneliu Sălișteanu*

- [12] J.M. Bergheau, R. Fortunier, *Finite Element Simulation of Heat Transfer*, John Wiley & Sons Inc., 2008