

CLOSED-LOOP MODEL ESTIMATION OF THE UNDERDAMPED SECOND ORDER INERTIAL SYSTEMS

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Abstract In this paper was proposed an approach for control-relevant identification. The experimental identification the mathematical model of the underdamped inertial systems is performed in the closed-loop and the control object is proposed to be approximated with transfer function with inertia second order, where the coefficients are calculated based on the simple analytical expressions according to the values extracted from underdamped step response of the closed-loop system with P controller. The proposed approach of control-relevant identification was verified by the computer simulation in software package MATLAB and the obtained results demonstrated, that the identification approach offers good results in process of estimation the model of the control object and methodology of tuning the PID controller offers possibility of imposing the performance to the automatic control system.

Keywords: control-relevant identification, experimental identification, closed-loop system identification, transfer function, second order underdamped inertial systems, PID controller.

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1. INTRODUCTION

Second-order inertial systems are widely used to model various dynamic processes in engineering and control systems as thermal process, flow process, mechanical systems with mass, damping, and stiffness elements. In electrical engineering, second-order systems can model RLC circuits, which consist of resistors, inductors, and capacitors. The mathematical model estimation can be realized by the analytical way, when are known theoretical principles, physical laws, and known relationships between variables from process, and by the experimental estimation. The experimental estimation involves collecting input/output data from the process and based on the experimental data using statistical and data-driven techniques are built models that describe the behavior of a process. The system estimation is depending on the goal of identification – analysis of the system or system control, according to these goals

the model estimation can be realized in the open-loop, or in the closed-loop [1]-[3].

Open-loop identification techniques prove to be straightforward to implement and there are a lot of methods that can be implemented in the time and frequency domains [12]. These methods are typically used when the dynamics of a process can be linearized around a nominal operating point, and the system is defined by parameters that remain constant over time. Numerous open-loop identification methods, such as Vítěčková's method, Latzel's method, Harriott's method, Smith's method, Strejc's method exist, that offer procedyres for accurately model estimation [9]-[11]. These methods are versatile, capable of handling systems with or without time delay and approximating systems with models of objects with inertia different order, ensuring a high level of precision in the identification process.

Over the past decade, there has been a resurgence of interest in closed-loop identification and control-relevant identification. Control-relevant identification is a specialized approach that tailors the identification process to extract information directly from the process relevant to the design and tuning the controller in real-time, ensuring that the identified models contribute effectively to achieving desired control objectives. Numerous control-relevant identification methods have been developed. These methods can be used with accuracy for implementation of auto-tuning controllers, that offers possibility in perturbation signal rejection and system performance ensurement. Regretably, a common limitation in the realm of closed-loop identification research is that the existing controller is linear and that the process is single-variable and linear [4]-[5].

In the present paper it is proposed an algorithm for control-relevant identification of the underdamped second order inertial systems in the closed-loop.

2. ALGORITHM FOR CLOSED-LOOP IDENTIFICATION OF THE UNDERDAMPED SECOND ORDER INERTIAL SYSTEM

In this paper is proposed an algorithm for model estimation of the underdamped second order inertial systems in the closed-loop, where the system is proposed to be approximated with the following transfer function:

$$H_F = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{k}{a_0 s^2 + a_1 s + a_2}, \quad (1)$$

where ω_n is natural frequency, ξ - damping ratio, k - transfer coefficient and

$$a_0 = \frac{1}{\omega_n^2}, a_1 = \frac{2\xi}{\omega_n}, a_2 = 1.$$

The block scheme of the closed-loop system is presented in Figure 1.

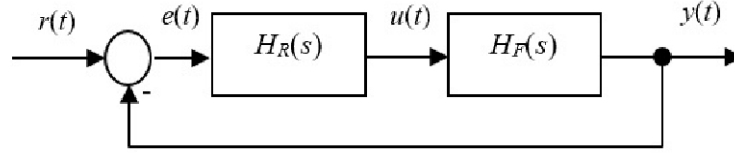


Fig. 1. Structural scheme of the automatic control system.

For model estimation is proposed to be used P controller with transfer function:

$$H_R(s) = k_p, \quad (2)$$

where k_p is proportional tuning parameter.

From the transfer function (1), the coefficients a_0, a_1, a_2 are proposed to be calculated based on the values of the k_p – proportional transfer coefficient of the P controller and T_0 – the period of oscillations, that are obtained for the case, when automatic control system generates the underdamped step response. To achieve this state of the system, it is considered to be given the closed-loop system with P controller (Figure 1) and the proportional tuning parameter $k_p > 0$ is varied, until the system achieves the underdamped step response presented in the Figure 2.

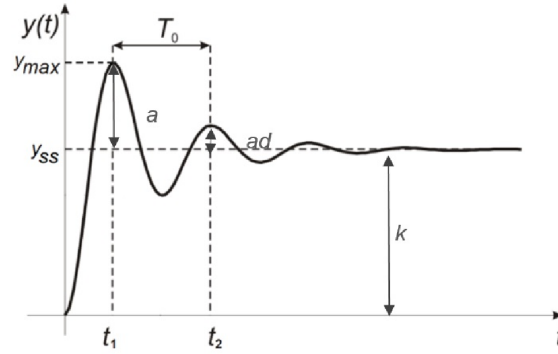


Fig. 2. Underdamped step response of the closed-loop system.

The static gain of the system is calculated by [6]:

$$k = \lim_{n \rightarrow \infty} \frac{\Delta y}{\Delta u} = \lim_{n \rightarrow \infty} \frac{y_{st} - y_{init}}{u - u_{init}}, \quad (3)$$

where y_{st} is the steady-state output value, $y_{initial}$ is the initial value of the output response, u – input signal, $u_{initial}$ is the initial value of the input signal.

The value of the damping ratio can be calculated based on the step response presented in Figure 2 as follows:

$$\xi = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\log d}\right)^2}}, \quad (4)$$

where d is decay ratio. Next, based on the underdamped step response of the closed-loop system, it is calculated the period of oscillations:

$$T_0 = t_2 - t_1. \quad (5)$$

If the values of period oscillations and damping ratio are known, then the value of natural frequency is calculated by [6]:

$$\omega_n = \frac{2\pi}{T_0 \sqrt{1 - \xi^2}}. \quad (6)$$

The closed-loop characteristic equation of the system with P controller is following:

$$A(s) = a_0 s^2 + a_1 s + a_2 + k k_p. \quad (7)$$

It is proposed the following substitution of the Laplace transform:

$$s = \omega_n j. \quad (8)$$

Based on (8) the characteristic equation (7) will become:

$$\begin{aligned} A(\omega_n j) &= (a_0(\omega_n j)^2 + a_1(\omega_n j) + a_2) + k k_p = -a_0 \omega_n^2 + a_2 + k k_p + a_1 \omega_n j = \\ &= P(\omega) + j Q(\omega). \end{aligned} \quad (9)$$

Next, it was set the real part equal to zero, denoted as $P(\omega) = 0$ and based on this equaling, there are obtained the following expressions for calculation the model parameters:

$$a_0 = \frac{k_{cr} k + a_2}{\omega_n^2}; a_2 = 1. \quad (10)$$

The coefficient a_1 remains unknown and for the calculation this coefficient it is used the maximum stability degree criterion [7]. According to this criterion,

the maximum displacement of the dominant poles of closed loop system to the imaginary axe in the left complex half plane is denoted by the η_{max} that are equal with:

$$\eta_{max} = \frac{a_1}{na_0}, \quad (11)$$

where n is order of the characteristic equation of the closed-loop system. From [7], it is known that

$$\eta_{max} = \omega_n \xi. \quad (12)$$

Equaling the expression (11) with (12), it can be obtained the expression for calculation the value of the a_1 coefficient:

$$a_1 = \frac{2(k_{cr}k + a_2)\xi}{\omega_n}. \quad (13)$$

In this way, according to the (3), (10) and (13) expressions, it can be determinate the values of coefficients k, a_0, a_1, a_2 from transfer function (1), that approximates the dynamics of the inertial process.

3. CONTROL-RELEVANT IDENTIFICATION ALGORITHM

One of the main goal of model identification is control algorithm synthesis so as to be achieved the imposed performance and ensured the perturbation signal rejection. Incorporating the model identification of the control object in the process of control algorithm design offers the potential to streamline the modelling process and this approach is known as control-relevant identification.

In the paper [8], it was proposed an algorithm for synthesis the PID controller to the second order inertial system with imposed performances, so as the control-relevant identification can be implemented in concordance with following algorithm:

1. Implementation of the feedback control system with P controller.
2. Variation of the proportional tuning parameter $k_p > 0$ until the system achieves underdamped step response, as is presented in the Figure 2.
3. From obtained underdamped step response presented in the figure 2 calculation the period of oscillations - T_0 .
4. From Figure 2 calculation the value of the damping ratio in concordance with expression (4).

5. Calculation of the system transfer coefficient according to the expression (3).
6. Calculation the value of natural frequency according to the expression (6).
7. Calculation the parameters of the transfer function $H_F(s)$ - (1):

$$a_0 = \frac{k_{cr}k + 1}{\omega_n^2}, a_1 = \frac{2(k_{cr}k + 1)\xi}{\omega_n}, a_2 = 1. \quad (14)$$

8. Imposing the performance to the automatic control system, namely the settling time - t_s .

9. Implementation the PID control algorithm in conformity with transfer function:

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s. \quad (15)$$

10. Tuning the PID controller in conformity with analytical expressions:

$$k_p = \frac{8(k_{cr}k + 1)\xi}{k\omega_n t_s}; \quad (16)$$

$$k_i = \frac{4}{k t_s}; \quad (17)$$

$$k_d = \frac{4(k_{cr}k + 1)}{k\omega_n^2 t_s}. \quad (18)$$

4. STUDY CASE OF CONTROL-RELEVANT IDENTIFICATION

For verification the proposed algorithm of control-relevant identification, it was supposed that the control object is described by the following transfer function:

$$H_F = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{4}{s^2 + 3.2s + 4} = \frac{1}{0.25s^2 + 0.8s + 1}. \quad (19)$$

The given system (19) was simulated in MATLAB with P controller, where the proportional tuning parameter k_p was varied until it was achieved the underdamped transient response of the closed loop system for $k_p = 6$.

The obtained underdamped step response is presented in Figure 3, from which there are calculated the period of oscillation $T_0 = 1.204s$. and damping ratio: $\xi = 0.3028$. The value of natural frequency was determinate by the expresion (6): $\omega_n = 5.4757$.

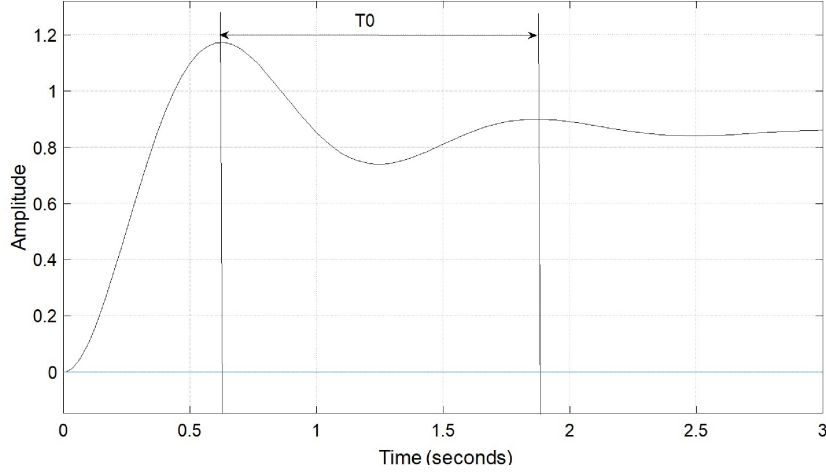


Fig. 3. Underdamped step response of the closed-loop system.

Next, according to the expressions (10) and (13), there were calculated the parameters of the identified transfer function:

$$a_0 = \frac{k_{cr}k + a_2}{\omega_n^2} = 0.2335; a_1 = \frac{2(k_{cr}k + a_2)\xi}{\omega_n} = 0.7742; a_2 = 1. \quad (20)$$

According to this calculations, the identified transfer function is following:

$$H_F = \frac{1}{0.2335s^2 + 0.7742s + 1}, \quad (21)$$

In Figure 4, it is presented the comparison between original step response of the open loop system described by the transfer function (19) – curve 1 and identified transfer function (21) – curve 2.

From Figure 4, it can be observed that the proposed methodology of model estimation in the closed-loop offered the goodness of fit 97%.

Next, according to the control-relevant identification algorithm it is proposed to be tuned the PID controller based on the expressions (16)-(18) for the case when it is imposed the value of settling time as $t_s = 1s, t_s = 2s$,

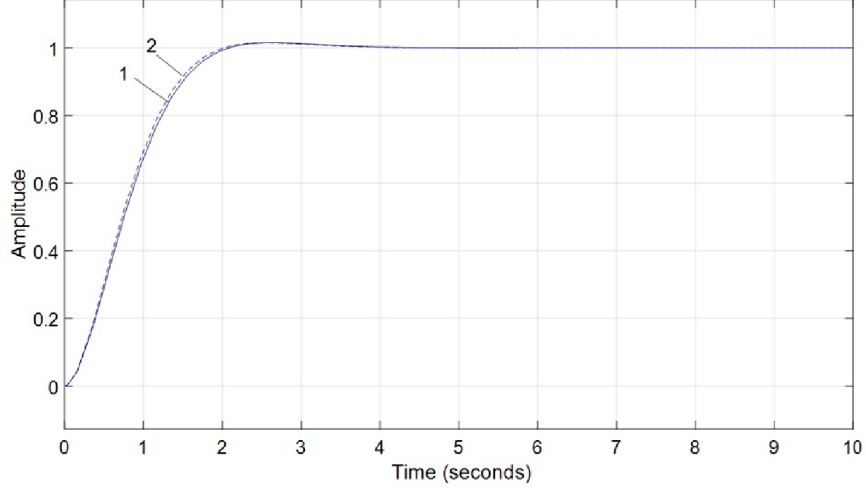


Fig. 4. Comparison of the system step responses in the open loop: 1-original transfer function (19); 2-identified transfer function (21).

$t_s = 4s, t_s = 60s$, the obtained results are presented in Table 1.

Table 1. The tuning parameters of PID controllers

No	Imposed settling time $t_s, s.$	k_p	k_i	k_d
1	1	3.0967	4	0.9339
2	2	1.5484	2	0.4669
3	4	0.7742	1	0.2335
4	10	0.3097	0.4	0.0934

The computer simulation of the automatic control system with PID controller is presented in the figure 5, where the curves numbering corresponds with numbering from Table 1.

In Table 2, there are presented the obtained performances of the automatic control system.

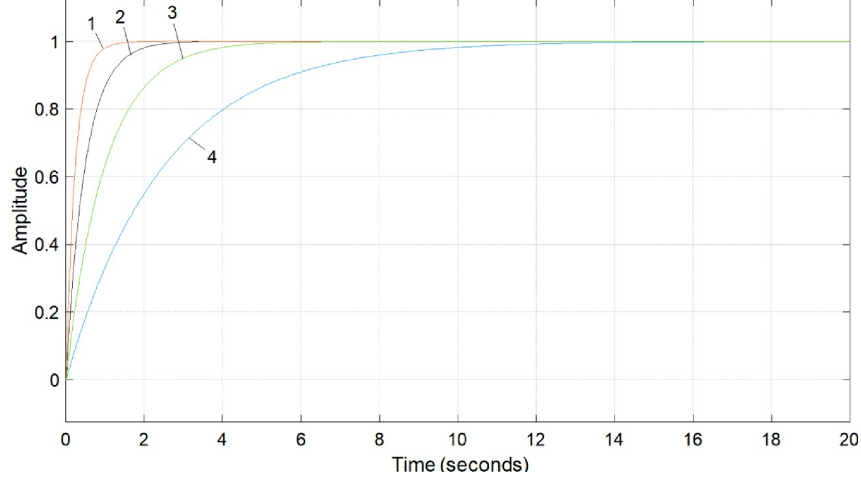


Fig. 5. Step responses of the automatic control system: 1 – PID controller with imposed settling time $t_s = 1s$; 2 – PID controller with imposed settling time $t_s = 2s$; 3 – PID controller with imposed settling time $t_s = 4s$; 4 – PID controller with imposed settling time $t_s = 10s$.

Table 2. The performances of the automatic control system

No	Settling time t_s, s	Rise time, t_r, s	Overshoot, σ
1	0.9779	0.9779	0
2	1.9559	1.9559	0
3	3.9121	3.9121	0
4	9.7804	9.7804	0

From Table 2 and Figure 5, it can be observed that the proposed methodology of tuning the PID controller to the model of object with inertia second order (21) offers high precision in tuning controller with imposed settling time - t_s , that was demonstrated through computer simulation.

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5. CONCLUSIONS

In the realm of control systems, traditional methodologies place a significant emphasis on the availability of accurate mathematical representations of the

system being controlled. In essence, the effectiveness of traditional control methods is closely tied to the ability to precisely capture and describe the dynamics of the system under consideration. For the last decades have been proposed and developed a lot of methods for models identification in the open-loop. These methods have disadvantage as can not be used for auto-tuning methods of PID controller.

In this paper, it is proposed the methodology for control-relevant identification, that supposes an algorithm for closed-loop model identification. Proposed method can be easily implemented in case of auto-tuning controllers, when it is necessary to be recalculate the mathematical model that approximates the dynamics of control object. There are presented simple analytical expressions for tuning the PID controller with imposed performances namely the settling time. The obtained results were verified by computer simulation and offer so high precision in model estimation with error 3% and in case of tuning the PID controller it was observed that system in totally satisfy the imposed performances.

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